GIRLS' HIGH SCHOOL AND COLLEGE,PRAYAGRAJ

2020 – 2021

CLASS - 12 B & C

MATHEMATICS

WORKSHEET NO. 7

CHAPTER: DIFFERENTIATIION

& MEAN VALUE THEOREM

Note: Parents are expected to ensure that the student spends two days to read and understand the topic according to the book and thereafter answer the questions.

Complete all the worksheet work in your math's register.

Book : ISC Mathematics for class 12 by OP Malhotra.

Continuation of Differentiation Chapter

Parametric Form : If the variables x and y are explicity expressed in terms of a third variable say x = f(t) and y = g(t), then x and y are called parametric functions or parametric equations and t is called the parameter.

Ex. If $x = \cos\theta - \cos 2\theta$ and $y = \sin\theta - \sin 2\theta$, then find dy/dx

Differentiating both x and y with respect to x

$\frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$; $\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$

Therefore, $\frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$

Logarithmic Differentiation : It is a technique to differentiate functions of the form $y = [u(x)]^{v(x)}$.

Ex. Differentiate a^x w.r.t x, where a is a positive constant.

Let $y = a^x$ taking log on both sides

$\log y = \log a^x$

$= x \log a$

Differentiating both the sides w.r.t x

$\frac{d(\log y)}{dx} = \frac{d}{dx}(x \log a)$

$\frac{1}{y}\frac{dy}{dx} = \log a$ $\qquad\qquad [x\frac{d(\log a)}{dx} + \log a\frac{dx}{dx}]$

$\frac{dy}{dx} = y \log a$

$= a^x \log a$

$\frac{d(a^x)}{dx}$ = $a^x loga$

Differentiation of a function u(x) with respect to another function v(x)

Ex : Find the derivative of sinx^2 with respect to x^2.

Let u = sinx^2 and v = x^2

Differentiating both the sides of u and v with respect to x

$\frac{du}{dx}$ = 2x cosx^2 and $\frac{dv}{dx}$ = 2x

Then, $\frac{du}{dv}$ = $\frac{du}{dx}$ / $\frac{dv}{dx}$

$\frac{du}{dv}$ = $\frac{2x\cos x^2}{2x}$ = cosx^2

Second Order Derivative: The second order derivative of a function is the derivative of the first order.

Let y = f(x) then y' = f '(x) i.e $\frac{\mathrm{dy}}{\mathrm{dx}}$ = f '(x)

$\frac{d}{dx}(\frac{dy}{dx})$ = $\frac{d}{dx}$[f '(x)]

$\Rightarrow$ $\frac{d^2y}{dx^2}$ = f ''(x)

Ex. y = sin(logx)

Differentiating it w.r.t. x

$\frac{dy}{dx}$ = cos(logx)$\frac{1}{x}$

Again differentiating it with respect to x

$\Rightarrow$ $\frac{d}{dx}[\frac{dy}{dx}]$ = $\frac{x[-\sin(logx)\frac{1}{x}]-\cos(logx)}{x^2}$

$\Rightarrow$ $\frac{d^2y}{dx^2}$ = $\frac{-[\sin(logx)+\cos(logx)]}{x^2}$

<u>**Exercise**</u>

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$

1. **1.** x = $\frac{1}{1-t^2}$, y = 1+ t^2
2. **2.** x = a $\cos^3$t , y = a $\sin^3$t

Differentiate:

1. $\tan^{-1}\frac{2x}{1-x^2}$ w.r.t. $\tan^{-1}$x
2. sinx^3 w..r.t $\sec^2 x^2$

Find the derivative of the following functions:

- $\sqrt{(x-1)(x-2)(x-3)(x-4)}$
- $(\tan x)^{\log x}$
- $(\log x)^x$
- If y = $x^{x^{x^{\cdots\cdots\infty}}}$, then show that $\frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$

Find the second derivative of the following functions:

1. logx
2. $\sin^{-1}x$
3. If y = ae^{mx}+ b e^{-mx}, prove that $\frac{d^2y}{dx^2}$ - $m^2y = 0$

MEAN VALUE THEOREM

Rolle's Theorem:

If function f(x) is (i) continuous in the closed interval [a,b] ,

(ii) differentiable in an open interval (a,b) and f(a) = f(b)

then there will be atleast one point c , where a < c < b such that

f'(c) = 0

Example:

Verify Rolle's Theorem for the function

f(x) = e^x(sinx – cosx) on $[\frac{\pi}{4}, \frac{5\pi}{4}]$

since e^x ,sinx and cosx are continuous functions in $[\frac{\pi}{4}, \frac{5\pi}{4}]$

so f(x) is continuous in $[\frac{\pi}{4}, \frac{5\pi}{4}]$

also, e^x ,sinx and cosx are differentiable in the $(\frac{\pi}{4}, \frac{5\pi}{4})$

Hence, f(x) is differentiable in $(\frac{\pi}{4}, \frac{5\pi}{4})$

Now, $f(\frac{\pi}{4}) = e^{\pi/4}(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}) = e^{\pi/4}(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}) = 0$

$f(\frac{5\pi}{4}) = e^{5\pi/4}(\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4})$

$= e^{5\pi/4}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 0$

Therefore, $f(\frac{\pi}{4}) = f(\frac{5\pi}{4})$

All three conditions are satisfied. So, there exists a point c$\in$ ($\frac{\pi}{4}$, $\frac{5\pi}{4}$), such that f'(c) = 0

Now, f'(x) = $e^x \frac{d}{dx}$(sinx – cosx) + (sinx – cosx) $\frac{d}{dx}$(e^x)

= e^x (cosx + sinx) + (sinx – cosx) e^x

= $2e^x$sinx

f '(x) = 0

$\Rightarrow$ $2e^x$sinc = 0

$\Rightarrow$ sinc = 0 [$e^x \neq 0$ for any value of x]

$\Rightarrow$ sinc = sin π since sin π =0

$\Rightarrow$ c = π

$\Rightarrow$ x = π $\in$ ($\frac{\pi}{4}$, $\frac{5\pi}{4}$)

Hence Rolle's theorem is satisfied.

Lagranges Mean Value Theorem

If f(x) is a function and is

i. continuous in the closed interval [a,b]
ii. derivable in the open interval (a,b)

Then there exists atleast one value c of x and c $\in$ (a,b) such that f'(c) = $\frac{f(b) - f(a)}{b-a}$

Example

Verify Lagranges mean value theorem for the function

f(x) = x(x-2) in [1,2]

The given function is a polynomial therefore it is continuous in [1,2] and derivable in (1,2)

f'(x) = 2x -2

f'(c) = 2c -2

$$\frac{f(b) - f(a)}{b-a} = \frac{f(2)-f(1)}{2-1}$$

$$= \frac{0 - (-1)}{1}$$

Since f'(c) = $\frac{f(b) - f(a)}{b-a}$

$\Rightarrow 2c - 2 = 1$

$\Rightarrow 2c = 3$

$\Rightarrow c = 3/2$

$3/2 \in (1,2)$

Therefore Lagranges mean value theorem is satisfied.

Exercise

Verify Rolles theorem

1. $f(x)=4\sin x$, $x \in [0,\pi]$

2. $f(x)= e^{x}\cos x$, $x \in [\frac{-\pi}{2},\frac{\pi}{2}]$

3. $f(x)= \sqrt{4-x^2}$ on$[-2,2]$

4. Apply Rolle's theorem to find point(or points)on the given curve $y = 16 - x^2$, $x\in[-1,1]$ where the tangent is parallel to the x axis.

5. Use Lagranges Mean Value Theorem to determine a point P on the curve $y =\sqrt{x-2}$

Defined in the interval [2,3] where the tangent is parallel to the chord joining the end points on the curve.

6. Verify Lagrange's Mean Value Theorem for the function

i. $f(x) =2x^2-10x +29$ in [2,7]

ii. $f(x) = \log x$ in [1,e]