GIRLS' HIGH SCHOOL AND COLLEGE

2020 – 2021

CLASS - 12 A&B

PHYSICS

WORKSHEET- 06

Chapter- ELECTRIC POTENTIAL

Topic – ELECTRIC POTENTIAL & ELECTRIC POTENTIAL ENERGY OF AN ELECTRIC DIPOLE

INSTRUCTIONS: Parents kindly ensure that your ward carefully goes through the assignment, its instructions and the chapter in the book, learns the subject matter and then answers the questions asked, in her register.

NOTE: Electric Potential is an important topic from the point of view of the ISC Examinations.

POINTS TO REVISE : Electric Field, Electric Potential, Relation between Field and Potential, Electric Potential due to multiple charges, Dipole, Dipole length and Dipole strength.

For any queries or questions you can mail me.

After revising the above topics answer the following.

Q1) The amount of work done in bringing a charge of 1.3×10^{-7} C

from infinity to a point in an electric field is 6.5×10^{-5} J. Find the electric potential at that point.

Q2) Find potential at a point due to a positive charge of 100 micro coulombs, at a distance of 9 m.

Q3) Two points A & B are 3 m apart. A point charge $q = 2 \times 10^{-2}$ C is placed at 'O' at a distance of 1 m from the point B on the line joining two charges in between A & B.

(i) Calculate the potential difference between A & B.

(ii) What will be the result if the positions of A & B are interchanged?

(iii) What will be the result if the point B is located at 1 m distance from 'O' perpendicular to the line joining OA?

Q4) What is an Electric Dipole? What are the factors on which its moment depends?

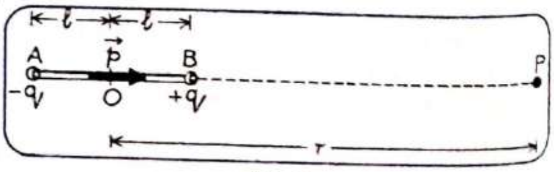
Now go through the following subject matter carefully and answer the questions that follow.

10 Potential due to an Electric Dipole

As we have read, an electric dipole is a pair of equal and opposite point charges, placed at a small distance. Its moment, known as electric dipole moment, is a vector $\vec{p}$ having a magnitude equal to the product of a charge and the distance between the charges, and a direction pointing from the negative to the positive charge. Let us determine electric potential due to a dipole at a point on its axial line, equatorial line and also at any point.

(i) Potential at a Point on the Axis of the Dipole

Let AB (Fig. 8) be an electric dipole formed by charges $-q$ and $+q$ coulomb, placed at a small distance $2l$ metre apart in vacuum (or air). Let P be a point along the dipole-axis at a distance r metre from the mid-point O of the dipole. We need to determine the electric potential at the point P. The distance of P from the charge $+q$ is $(r-l)$ and that from the charge $-q$ is $(r+l)$. Therefore, the potential at P due to the charge $+q$ of the dipole is



(Fig. 8)

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)}$$

and that due to the charge $-q$ is

$$V_2 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l)}.$$

Electric potential is a scalar quantity. Hence, the resultant potential V at the point P will be the algebraic sum of the potentials V_1 and V_2; that is

$$V = V_1 + V_2$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-l)} - \frac{1}{(r+l)}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+l)-(r-l)}{r^2-l^2}\right] = \frac{q}{4\pi\varepsilon_0}\frac{2l}{r^2-l^2} = \frac{1}{4\pi\varepsilon_0}\frac{2ql}{r^2-l^2}.$$

But $2ql = p$ (electric dipole moment).

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2-l^2}.$$

If r is very large as compared to $2l$ ($r >> 2l$), then l^2 can be neglected in comparison to r^2. Then, the potential at the point P due to the electric dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \text{ volt.}$$

ii) Potential at a Point on the Equatorial Line of the Dipole

Now, suppose that the point P is situated on the equatorial line of the dipole AB at a distance r metre from its mid-point O (Fig. 9). The potential at P due to the charge $+q$ of the electric dipole is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP}$$

and that due to the charge $-q$ is

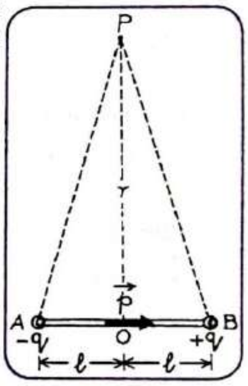
$$V_2 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{AP}.$$

$\therefore$ Resultant potential at P is

$$V = V_1 + V_2$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{BP} - \frac{1}{AP}\right)$$

$$= 0. \qquad (\because BP = AP)$$



(Fig. 9)

Thus, the electric potential is everywhere zero on the equatorial line of a dipole (but intensity is not zero). No work is done in moving a charge along this line.

PLEASE NOTE : Above case is an example of a situation where field is not zero but potential is zero.

Q5) Derive an expression for Potential at an Axial position of an Electric Dipole. Compare the Electric Field for the above position with the Electric Potential particularly in terms of their variation with distance from dipole.

Q6) Similarly derive an expression for Potential at a point on the equatorial line of a dipole and compare it with the field at the same point for the same dipole.

Q7) A and B are two points on the axis and the perpendicular bisector respectively of an electric dipole and at equal distances from it. The fields at A and B are $\vec{E}_A$ and $\vec{E}_B$. Compare $\vec{E}_A$ and $\vec{E}_B$.

Let V_A and V_B be potentials at A and B respectively then compare V_A and V_B also.

Q8) Two charges of ±3 μC are at a distance of 3×10^{-3} m from each other. Calculate (i) electric potential at a distance of 0.6 m from the dipole in broad- side- on position, (ii) electric potential at the same point after rotating the dipole through 90°.

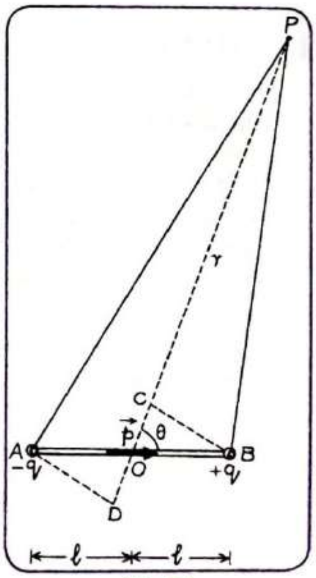
Q9) Calculate the electric potential at the surface of the nucleus of a silver atom. The radius of the nucleus is 3.4×10^{-14} m and the atomic number of silver is 47.

iii) Potential at any Point

Let P (Fig. 10) be a point at a large distance r from the dipole at which electric potential is required. Let (r, θ) be the polar coordinates of P. Let us join PA and PB and draw AD and BC perpendiculars to OP. Since, $r >> l$, we can write

$$BP = CP = OP - OC = r - l\cos\theta$$

and $$AP = DP = OP + OD = r + l\cos\theta.$$



(Fig. 10)

Now, the potential at P due to the charge $+q$ of the dipole is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r - l\cos\theta)}$$

and that due to the charge $-q$ is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{-q}{AP} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r + l\cos\theta)}.$$

The resultant potential (a scalar) at P is

$$V = V_1 + V_2$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r - l\cos\theta)} - \frac{1}{(r + l\cos\theta)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(r + l\cos\theta) - (r - l\cos\theta)}{r^2 - l^2\cos^2\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{2l\cos\theta}{(r^2 - l^2\cos^2\theta)} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{(r^2 - l^2\cos^2\theta)}$$

where $p\ (= 2ql)$ is the magnitude of the dipole moment. Because $r >> l$, $l^2\cos^2\theta$ may be neglected in comparison to r^2. Then, the resultant potential at point P is

$$\mathbf{V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}\ \text{volt.}}$$

We can see from this general result that, $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$ for an axial point $(\theta = 0°)$

and $V = 0$ for a point on equatorial line $(\theta = 90°)$.

In vector form : $$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{r}}}{r^3}.$$

Comparing this result with the potential due to a point charge, we see that :

(i) in a fixed direction, that is, fixed θ, $V \propto 1/r^2$ here rather than $V \propto 1/r$.

(ii) even for a fixed distance r, there is now a dependence on direction, that is, on θ.

Difference between Electric Potential at a Point due to Single Point Charge and an Electric Dipole

1. Potential at a point due to single point charge depends only upon the distance r while due to an electric dipole depends upon the distance r and inclination θ of r.
2. Potential due to a single point charge varies as $V \propto \frac{1}{r}$ while potential due to dipole varies as $V \propto \frac{1}{r^2}$.
3. Potential due to a single point charge has a spherical symmetry about the point of observation, while potential due to an electric dipole has cylindrical symmetry about the electric dipole axis.

Q10) Obtain a general expression for electric potential at any point in the vicinity of an electric dipole. Write the expression in vector form and obtain potentials at end- on and broad- on positions as special cases.

IMPORTANT QUESTION

Q11) What are the differences between electric potential at a point, due to a single point charge and an electric dipole.

IMPORTANT NOTE : Recall the derivation of the expression for torque on a dipole in a uniform electric field and how work done is related to torque in rotational motion. Go through your book for these topics thoroughly and answer the following question.

Q12) An electric dipole of charges $\pm 0.6 \times 10^{-19}$ C, 0.3 mm apart, when held at 45° with respect to a uniform electric field of

10^4 NC^{-1},experiences a torque. What is its value?

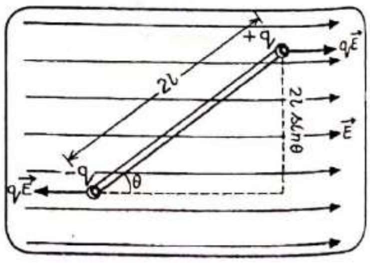
Continuing the same topic as above we will extend it to find work done and energy stored in a dipole. All these topics are quite important so you are expected to do them again and again till you are thorough with it.

Now go through the following derivation

11 Work Done in Rotating an Electric Dipole in an Electric Field

If a dipole placed in a uniform electric field is rotated from its equilibrium position, work has to be done. Suppose, an electric dipole of moment p (= $q \times 2l$) is rotated in a uniform electric field E through an angle θ from its equilibrium position. During rotation, the couple acting on the dipole changes. Suppose, at any instant, the dipole makes an angle θ with the electric field E (Fig. 11). Then, the instantaneous torque acting on the dipole is

$$\tau = p\,E \sin\theta.$$



(Fig. 11)

The work done in rotating the dipole further from this position through an infinitesimally small angle $d\theta$ is

$$dW = \text{torque} \times \text{angular displacement} = \tau\, d\theta = p\,E \sin\theta\, d\theta .$$

Hence, the work done in rotating the dipole from the angle θ_1 to θ_2

$$W = \int_{\theta_1}^{\theta_2} p\,E \sin\theta\, d\theta = p\,E\left[-\cos\alpha\right]_{\theta_1}^{\theta_2} = p\,E\,[-\cos\theta_2 + \cos\theta_1]$$

or

$$\mathbf{W = pE\,(\cos\theta_1 - \cos\theta_2).}$$

If the dipole is rotated from its equilibrium position ($\theta = 0$) to angle θ with the field

$$W = pE[1 - \cos\theta]$$

This is the formula for the work done in rotating an electric dipole in a uniform electric field through an angle θ from the direction of the field (equilibrium position) an shown in (Fig. 11).

If the dipole be rotated through 90° from the direction of the field, ($\theta = 90°$) then the work done will be

$$W = p\,E\,(1 - \cos 90°) = p\,E\,(1 - 0) = p\,E.$$

Similarly, if the dipole be rotated through 180° from the direction of the field, then the work done will be

$$W = p\,E\,(1 - \cos 180°) = p\,E\,[1 - (-1)] = 2\,p\,E.$$

Q13) An electric dipole of moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$, with $\vec{p}$ parallel to $\vec{E}$. It is then rotated by an angle θ. Find the work done. What is the equilibrium position of this dipole? What is the work done if the dipole is rotated through a) 90° b) 120° ?

Q14) Two point charges of ± 1.0×10^{-6} C are at a distance 2.0 cm from each other. This dipole is situated in a uniform electric field of 1.0×10^{5} Vm^{-1}. What will be the maximum torque acting on it due to the field? What will be the work done in rotating it through 180° from the equilibrium position?

Q15) An electric dipole of length 2.0 cm is placed with its axis making an angle of 60° with a uniform electric field of 10^{5} NC^{-1}. If it experiences a torque of

$8\sqrt{3}$ N m, calculate (i) magnitude of charge on the dipole

(ii) potential energy of the dipole.

Q16) The distance between two protons is 1.0×10^{-10} m. If they are made free, what will be the Kinetic energy of each when they are infinite distance apart? If one proton is kept fixed and only other is free then what are their new kinetic energies?

Q17) Two electrons are released towards each other with equal velocities of 10^6 ms^{-1}. What will be the closest approach between them?

Q8) Calculate the voltage needed to balance an oil drop carrying 10 electrons between two conductor plates of a capacitor 5 mm apart. The mass of the drop is 3×10^{-16} kg and g = 10 ms^{-1}.

Now note again that the potential energy stored in a system is the net work done either on the system or in building the system in the existing force field. You have already done expression for electric potential energy in case of a system of charges. Please revise.

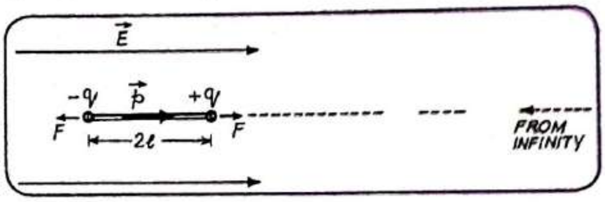
Q19) Three equal charges of 5.0 μC each are placed at the three vertices of an equilateral triangle of side 5.0 cm. Calculate electrostatic potential energy of the system of charges.

Now we apply the same fundamental in finding potential energy of an electric dipole.

Electric Potential Energy of an Electric Dipole in an Electrostatic Field

The potential energy of an electric dipole in an electric field is equal to the work done in bringing the dipole from infinity to inside the field.

In Fig. 12, an electric dipole is brought from infinity to a uniform electric field E in such a way that the dipole moment p is always in the direction of the field. Due to the field E, a force F (= qE) acts on the charge $+q$ in the direction of the field, and an equal force F (= qE) on the charge $-q$ in the opposite direction. Hence, in bringing the dipole in the field, work will be done on the charge $+q$ by an external agent, while work will be done by the field itself on the charge $-q$. But, as is clear from the Fig. 12, as the dipole is brought from infinity into the field, the charge $-q$ covers $2l$ distance more than the charge $+q$. Therefore, the work done on $-q$ will be greater. Hence, the 'net' work done in bringing the dipole from infinity into the field



(Fig. 12)

$$= \text{force on charge } (-q) \times \text{additional distance moved}$$
$$= -qE \times 2l = -pE,$$

where p (= $q \times 2l$) is the moment of the electric dipole. This work is the potential energy U_0 of the electric dipole placed in the electric field parallel to it :

$$U_0 = -pE.$$

In this position the electric dipole is in 'stable' equilibrium inside the field.

Now, if we rotate the dipole in the field through an angle θ, then work will have to be done on the dipole. This work is given by

$$W = pE(1 - \cos\theta).$$

This will result in an increase in the potential energy of the dipole. Hence, the potential energy of the dipole in the position θ will be given by

$$U_\theta = U_0 + W = -pE + pE(1 - \cos\theta)$$

or $$U_\theta = -pE\cos\theta.$$

This is the general equation of the potential energy of the electric dipole.

In vector notation :

$$U = -\vec{p} \cdot \vec{E}.$$

Particular Cases : (i) If the dipole is perpendicular to the field ($\theta = 90°$), then

$$U_{90°} = -pE\cos 90° = 0.$$

In this position the potential energy of the dipole is zero. This can be explained in another way also. If we keep the dipole perpendicular to the field while bringing it from infinity into the field, then the work done on the charge $+q$ by the external agent will be equal to the work done on the charge $-q$ by the field. Thus, the net work done on the dipole will be zero and hence the potential energy of the dipole will also be zero.

(ii) When the dipole moment $\vec{p}$ is in the direction of electric intensity ($\vec{E}$), *i.e.*, $\theta = 0$. The dipole is called in the condition of stable equilibrium. In this position, its potential energy

$$U_0 = -pE\cos 0 = -pE \text{ (least)}$$

(iii) If the dipole be rotated through 180° from the position of stable equilibrium, then the potential energy in the new position will be

$$U_{180°} = -pE\cos 180° = +pE.$$

In this position the dipole will be in 'unstable' equilibrium.

Q20) Derive an expression for potential energy stored in an electric dipole oriented in an electric field in the vector form.

END