GIRLS' HIGH SCHOOL & C0LLEGE, PRAYAGRAJ

WORKSHEET: 07

SESSION: 2020-21

CLASS: IX A, B, C, D,E & F

SUBJECT: MATHEMATICS

INSTRUCTIONS: Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and websites referred and thereafter answer the given questions.

NOTE: **1**. Students should refer to books related to class 6,7 &8 for reference and also the following websites www.extramarks.com and www.topperlearning.com

2. Concise Mathematics I.C.S.E. CLASS- IX by R.K. BANSAL

3.Understanding I.C.S.E. Mathematics CLASS- IX by M.L. AGGARWAL

T0PIC : LOGARITHMS

Logarithms are used to make the long and complicated calculations easy.

Consider $3^4 = 81$, this is the exponential form of representing relation between three numbers 3,4,81 .Now the same relation between 3,4 and 81 can be written as

$\log_3 81 = 4$ (read as: **logarithm of 81 at base 3 is 4**)

Thus, $3^4 = 81 \Leftrightarrow \log_3 81 = 4$

Definition: If a, b and c are three real numbers such that $a \neq 1$ and $a^b = c$ then b is called logarithm of c at the base a and is written as $\log_a c = b$; read as log of c at the base a is b.

$$a^b = c \Leftrightarrow \log_a c = b$$

INTERCHANGING

(logarithmic form vis-à-vis exponential form)

$a^b = c$ is called the exponential form

and, $\log_a c = b$ is called the logarithmic form

i.e., i) $2^{-3} = 0.125$ [exponential form]

$\Rightarrow$log of 0.125 to the base 2 = -3

i.e., $\log_2 0.125 = -3$ [logarithmic form]

ii) $\log_{64} 8 = \frac{1}{2}$ [logarithmic form]

$\Rightarrow$log of 8 to the base 64 $= \frac{1}{2}$

i.e., $(64)^{1/2} = 8$ [exponential form] and so on.

similarly:

If x is positive;

iii) $x^0 = 1 \Rightarrow \log_x 1 = 0$ i.e. log of 1 to the base x =0

In general; **the logarithm of 1 to any base is zero.**

i.e. $\log_5 1 = 0$; $\log_{10} 1 = 0$; $\log_a 1 = 0$ and so on.

iv) $x^1 = x \Rightarrow \log_x x = 1$ i.e. log x to the base x =1

In general**, the logarithm of any number to the same base is always** 1.

i.e. $\log_5 5 = 1$; $\log_{10} 10 = 1$; $\log_a a = 1$ and so on

SOLVE THE FOLLOWING QUESTIONS ACCORDING TO THE EXAMPLES GIVEN

Example 1:

Find i) the logarithm of 1000 to the base 10

ii)the logarithm of 1/9 to the base 3

Solution :i) Let $\log_{10} 1000 = x$

$10^x = 1000$

$10^x = 10^3$

$\Rightarrow x = 3$

ii) Let $\log_3 1/9 = x$

$3^x = 1/9$

$3^x = 3^{-2}$

$\Rightarrow x = -2$

Example 2:

Find x, i) $\log_2 x = -2$

ii) $\log_4(x+3) = 2$

iii) $\log_x 64 = 3/2$

Solution:i) $\log_2 x = -2$

$2^{-2} = x$

$x=1/4$

ii)) $\log_4(x+3) =2$

$4^2 = x+3$

$16 = x+3$

$x=13$

iii) $\log_x 64 = 3/2$

$x^{3/2} =64$

$x^{3/2} =2^6$

$x = 2^{6x2/3}$

$x = 2^4$

$x= 16$

Question 1: Express each of the following in logarithmic form:

i) $5^3 =125$
ii) $3^{-2}= 1/9$
iii) $10^{-3} =0.001$
iv) $(81)^{3/4} = 27$

Question 2: Express each of the following in exponential form:

i)$\log_8 0.125 = -1$

ii) $\log_{10} 0.01 =-2$

Question 3: Find log of :

i) 100 to the base 10

ii) 0.1 to the base 10

iii)32 to the base 4

iv)1/16 to the base 4

v)1/81 to the base 27

Question 4: Find x, if :

i) $\log_3 x=0$
ii) $\log_9 243=x$
iii) $\log_5(x-7)=1$
iv) $\log_7(2x^2-1)=2$

Question 5: Evaluate:

i) $\log_2(1 \div 8)$
ii) $\log_5 1$
iii) $\log_{16} 8$

Example 3:

If $\log_a m=n$, express a^{n-1} in terms of a and m

Solution: $\log_a m=n$

$a^n=m$

$a^{n-1}=a^n/a$

$=m/a$

Question 6:Given $\log_2 x = m$ and $\log_5 y = n$

i)Express 2^{m-3} in terms of x

ii)Express 5^{3n+2} in terms of y

Question 7: If log $(x^2 - 21) = 2$, show that $x = \pm 11$

LAWS OF LOGARITHM

1) PRODUCT LAW : The logarithm of a product at any non zero base is equal to the sum of the logarithms of its factors at the same base

i.e. $\log_a (mxn) = \log_a m + \log_a n$

$\log_x (m \times n \times p) = \log_x m + \log_x n + \log_x p$ and so on

Remember : $\log_a (m+n) \neq \log_a m + \log_a n$

2) QUOTIENT LAW: The logarithm of a fraction at any non zero base is equal to the difference between the logarithm of the numerator minus the logarithm of the denominator, both at the same base

i.e. $\log_a m/n = \log_a m - \log_a n$

Remember: $\log_a m/\log_a n \neq \log_a m - \log_a n$. Also, $\log_a (m-n) \neq \log_a m - \log_a n$

3) POWER LAW:

The logarithm of a power of a number at any non zero base is equal to the logarithm of the number (at the same base) multiplied by the power

i.e. $\log_a (m)^n = n\log_a m$

Corollary: since $\sqrt[n]{m} = m^{1/n}$

Therefore, $\log_a \sqrt[n]{m} = \log_a m^{1/n} = 1/n \log_a m$

NOTE:

1)logarithms to the base 10 are known as common logarithms.

2)If no base is given, the base is always taken as 10

i.e. $\log 8 = \log_{10} 8$; $\log a = \log_{10} a$; $\log 10 = \log_{10} 10$ and so on

3) $\log_{10} 1 = 0$; $\log_{10} 10 = 1$;

$\log_{10} 100 = 2$ $[\log_{10} 100 = \log_{10} 10^2 = 2\log_{10} 10 = 2x1 = 2]$

Similarly, $\log_{10} 1000 = 3$; $\log_{10} 10000 = 4$ and so on

EXPANSION OF EXPRESSIONS WITH THE HELP OF LAWS OF LOGARITHM

Let $y = \frac{a^4 \times b^2}{c^3} \Rightarrow \log y = \log \frac{a^4 \times b^2}{c^3}$

i.e. $\log y = \log (a^4 \text{ x } b^2) - \log c^3$ $[\log \frac{m}{n} = \log m - \log n]$

$= \log a^4 + \log b^2 - \log c^3$

$= 4\log a + 2\log b - 3\log c$

$\log y = 4\log a + 2\log b - 3\log c$ is the logarithmic expansion of the given expression $y = \frac{a^4 \times b^2}{c^3}$

similarly,

$m = \frac{3^x}{5^y \times 8^z} \Rightarrow \log m = \log 3^x - \log (5^y \times 8^z)$

$=x\log 3 - [\log 5^y + \log 8^z]$

$=x\log 3 - y\log 5 - z\log 8$

Conversely:

$\log V = \log\pi + 2\log r + \log h - \log 3$

$=\log\pi + \log r^2 + \log h - \log 3$

$=\log\frac{\pi r^2 h}{3}$

$V=\frac{\pi r^2 h}{3}$

Example 4:

Express $\log_{10}\sqrt[5]{108}$ in terms of $\log_{10}2$ and $\log_{10}3$

Solution: $\log_{10}\sqrt[5]{108} = \log_{10}(108)^{1/5}$ [$\sqrt[n]{m}=m^{/n}$]

$=\frac{1}{5}\log_{10}108$ [$\log_{10}n^m=m\log_{10}n$]

$=\frac{1}{5}\log_{10}(2^2x3^3)$ [$108=2x2x3x3x3=2^2x3^3$]

$= \frac{1}{5}[\log_{10}2^2+\log_{10}3^3]$ [$\log_{10}mxn=\log_{10}m+\log_{10}n$]

$=\frac{1}{5}[2\log_{10}2 + 3\log_{10}3]$

Example 5:

Express as a single logarithm: $2+\frac{1}{2}\log_{10}9 - 2\log_{10}5$

Solution: $2+\frac{1}{2}\log_{10}9 - 2\log_{10}5$

$=\log_{10}100 + \log_{10}9^{1/2} - \log_{10}5^2$ [$\log_{10}100=2;\log 9=\log 3^2$;

$2\log 5 = \log 5^2]$

$= \log_{10}100 + \log_{10}3 - \log_{10}25$

$= \log_{10}\frac{100\times 3}{25}$

$= \log_{10}12$ $\quad$ $[\log a + \log b - \log c = \log\frac{a\times b}{c}]$

Example 6:

Find x,if (i) $\log_{10}(x+5) = 1$

ii) $\log_{10}(x+1) + \log_{10}(x-1) = \log_{10}11 + 2\log_{10}3$

Solution:i) $\log_{10}(x+5) = 1$

$\log_{10}(x+5) = \log_{10}10$

$\Rightarrow x+5 = 10$

$x = 5$

ii) $\log_{10}(x+1) + \log_{10}(x-1) = \log_{10}11 + 2\log_{10}3$

$\log_{10}(x+1)(x-1) = \log_{10}11 + \log_{10}3^2$

$\log(x^2-1) = \log(11 \text{ x } 9)$

$\Rightarrow (x^2-1) = 99$

$x^2 = 100$

$x = 10$

Question 8 :Express each of the following in a form free from logarithm:

i) $2\log x - \log y = 1$

ii) 2log x + 3 logy = log a

iii)a logx –b log y =2log3

Question 9: Evaluate each of the following without using tables:

i)log 5+log 8 – 2log2

ii)$\log_{10}8 + \log_{10}25 + 2\log_{10}3 - \log_{10}18$

iii)$\log 4 + \frac{1}{3}\log 125 - \frac{1}{5}\log 32$

Question10 :Prove that:

$2\log\frac{15}{18} - \log\frac{25}{162} + \log\frac{4}{9} = \log 2.$

THE END