GIRLS' HIGH SCHOOL & C0LLEGE, PRAYAGRAJ

WORKSHEET: 06

SESSION: 2020-21

CLASS: IX A, B, C, D, E ,F

SUBJECT: MATHEMATICS

INSTRUCTIONS: Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and websites referred and thereafter answer the given questions.

NOTE: 1. Students should refer to books of class 6, 7 & 8 for reference and also the following websites:" www.extramarks.com" and "www.topperlearning.com"

2 .Concise MATHEMATICS I.C.S.E. CLASS- IX by R.K. BANSAL

3. Understanding I.C.S.E. MATHEMATICS CLASS- IX by M.L. AGGARWAL

TOPIC:- INDICES [EXPONENTS]

Exponent- If a is a real number and n is an integer, we know

a x a x a x a x a x…….n times = a^n ,where a^n is called an exponential expression with base a and exponent (or index or power) n.

a^n is read as **a raised to the power n**.

LAWS OF EXPONENTS

1) **Product law** : $a^m \times a^n = a^{m+n}$
   e.g. : $3^5 \times 3^4 = 3^{5+4} = 3^9$

2) **Quotient law** : $\frac{a^m}{a^n}$ = a^{m-n}

e.g. : $\frac{2^{12}}{2^7} = 2^{12-7} = 2^5$

3) **Power law** : $(a^m)^n = a^{mn}$

e.g. : $(3^5)^2 = 3^{5x2} = 3^{10}$

$(7^{-2})^3 = 7^{-2 \times 3} = 7^{-6}$

NOTE: i) if n is even, $(-a)^n$ is positive.

ii) if n is odd, $(-a)^n$ is negative.

e.g. $(-2)^3$ = -2 x -2 x -2 =-8

$(-2)^4$ = -2 x -2 x-2 x-2 = 16

$(-2)^5$ = -2 x -2 x -2 x -2 x -2 = -32

4) For any non zero rational number a

$a^{-n} = \frac{1}{a^n}$ and $a^n = \frac{1}{a^{-n}}$

i.e. a^{-n} and a^n are reciprocal of each other

e.g. i) $5^{-3} = \frac{1}{5^3}$

ii) $7^3 = \frac{1}{7^{-3}}$

MORE ABOUT EXPONENTS

1) $(a \times b)^n = a^n \times b^n$

2) $(\frac{a}{b})^n = \frac{a^n}{b^n}$

3) $a^0 = 1$; if $a \neq 0$

4) $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$; $a \neq 0$

5) $\sqrt[n]{a} = a^{1/n}$ and $\sqrt[n]{a^m} = a^{m/n}$

SOLVE THE FOLLOWING QUESTIONS ACCORDING TO THE EXAMPLES GIVEN

Example 1:

Evaluate: $27^{-1/3}$

Solution : $27^{-1/3} = 3^{3 \times -1/3}$

$= 3^{-1}$

$= \frac{1}{3}$

Example 2:

Evaluate: $9^{3/2} - 3.(5)^0 - (\frac{1}{81})^{-1/2}$

Solution : $9^{3/2} - 3.(5)^0 - (\frac{1}{81})^{-1/2}$

$= 3^{2 \times 3/2} - 3 \times 1 - (81)^{1/2}$

$= 3^3 - 3 - 9^{2 \times ½}$

$= 27 - 3 - 9$

$= 15$

Example 3:

Evaluate : $[5(8^{1/3} + 27^{1/3})^3]^{1/4}$

Solution : $[5(8^{1/3} + 27^{1/3})^3]^{1/4}$

$= [5(2^{3 \times 1/3} + 3^{3 \times 1/3})^3]^{1/4}$

$= [5(2+3)^3]^{1/4}$

$= (5 \times 5^3)^{1/4}$

$= 5^{4 \times 1/4}$

$=5$

Example 4:

Given : $1176 = 2^p . 3^q . 7^r$, find

i) The numerical values of p, q and r.

ii) The value of $2^p . 3^q . 7^{-r}$ as a fraction

Solution : i)1176 = 2x2x2x3x7x7

$=2^3 . 3^1 . 7^2$

Therefore, p=3,q=1 and r=2.

ii) $2^3 . 3^1 . 7^{-2} = \frac{8\times3}{7^2}$

$=\frac{24}{49}$

Example 5:

Simplify: $\frac{3^{a+2} - 3^{a+1}}{4 \times 3^a - 3^a}$

Solution: $\frac{3^a . 3^2 - 3^a . 3^1}{4\times3^a - 3^a}$

$=\frac{3^a (3^2 - 3^1)}{3^a (4-1)}$

$=\frac{9-3}{3}$

$= 2$

Example 6:

Simplify: $(\frac{a^m}{a^n})^{m+n} \cdot (\frac{a^n}{a^l})^{n+l} \cdot (\frac{a^l}{a^m})^{l+m}$

Solution : $(a^{m-n})^{m+n} \cdot (a^{n-l})^{n+l} \cdot (a^{l-m})^{l+m}$

$= a^{m^2-n^2} \cdot a^{n^2-l^2} \cdot a^{l^2-m^2}$

$= a^{m^2-n^2+n^2-l^2+l^2-m^2}$

$= a^0$

$= 1$

Question 1: Solve the following:

i) $3^3 \times (243)^{-2/3} \times 9^{-1/3}$

ii) $7^0 \times (25)^{-3/2} - 5^{-3}$

iii) $(\frac{16}{81})^{-3/4} \times (\frac{49}{81})^{3/2} \div (\frac{343}{216})^{2/3}$

iv) $(8x^3 \div 125y^3)^{2/3}$

Question 2: Simplify-

i) $(a + b)^{-1} \cdot (a^{-1} + b^{-1})$

ii) $\frac{5^{n+3} - 6\times5^{n+1}}{9\times5^n - 5^n\times2^2}$

iii) $(3x^2)^{-3} \times (x^9)^{2/3}$

iv) $\frac{3\times27^{n+1} + 9\times3^{3n-1}}{8\times3^{3n} - 5\times27^n}$

Question 3: If $2160 = 2^a \cdot 3^b \cdot 5^c$,find a,b and c. Hence, calculate the value of $3^a \times 2^{-b} \times 5^{-c}$.

Question 4:

i)Simplify :$[1- \{1-(1-n)^{-1}\}^{-1}]^{-1}$

ii) Show that $(\frac{a^m}{a^{-n}})^{m-n} \cdot (\frac{a^n}{a^{-l}})^{n-l} \cdot (\frac{a^l}{a^{-m}})^{l-m} = 1$

iii)Simplify: $(\frac{x^a}{x^b})^{a^2+ab+b^2} \cdot (\frac{x^b}{x^c})^{b^2+bc+c^2} \cdot (\frac{x^c}{x^a})^{c^2+ca+a^2}$

Solving exponential equations

For solving an exponential equation, express both of its sides into terms with the same base. Then, the exponents on both the sides of the equation are equal

i.e. if $a^x = a^y \Rightarrow x=y$.

Example 1:

Solve for x:

i) $9 \cdot 3^x = (27)^{2x-5}$

ii) $\sqrt{(\frac{3}{5})^{1-2x}} = 4\frac{17}{27}$

Solution: i) $9 \cdot 3^x = (27)^{2x-5}$

$$3^2 \cdot 3^x = (3^3)^{2x-5}$$

$$3^{2+x} = 3^{6x-15}$$

i.e. $2+x = 6x - 15$

$$x=\frac{17}{5}$$

$$x=3\frac{2}{5}$$

ii) $\sqrt{(\frac{3}{5})^{1-2x}} = 4\frac{17}{27}$

$[(\frac{3}{5})^{1-2x}]^{1/2} = \frac{125}{27}$

$(\frac{3}{5})^{\frac{1-2x}{2}} = (\frac{5}{3})^3$

$(\frac{3}{5})^{\frac{1-2x}{2}} = (\frac{3}{5})^{-3}$

i.e. $\frac{1-2x}{2} = -3$

x = 3.5

Example 2:

Solve: $2^{2x+3} - 9 \times 2^x + 1 = 0$

Solution: $2^{2x} \times 2^3 - 9 \times 2^x + 1 = 0$

$8y^2 - 9y + 1 = 0$ [taking $2^x = y$]

$8y^2 - 8y - y + 1 = 0$

$(8y - 1)(y - 1) = 0$

i.e. y = 1/8 or 1

when y=1/8 $\Rightarrow 2^x = 2^{-3} \Rightarrow x = -3$

when y= 1 $\Rightarrow 2^x = 2^0 \Rightarrow x = 0$

Example 3:

Prove that : $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1$

Solution : L.H.S. = $\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$

$= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a}$

$= \frac{x^a+x^b+x^c}{x^a+x^b+x^c}$

=1=R.H.S.

Example 4:

If $a=b^{2x}$, $b=c^{2y}$ and $c= a^{2z}$,show that 8xyz=1.

Solution : $a = b^{2x}$ and $b=c^{2y} \Rightarrow a = (c^{2y})^{2x} = c^{4xy}$

Similarly, $a=c^{4xy}$ and $c=a^{2z} \Rightarrow a = (a^{2z})^{4xy} = a^{8xyz}$

Now, $a=a^{8xyz} \Rightarrow 8xyz=1$

Question 5: Solve for x:

i) $2^{2x+1}=8$

ii) $3^{4x+1}=(27)^{x+1}$

iii) $(49)^{x+4}=7^2 \,.(343)^{x+1}$

iv) $(\sqrt{\frac{3}{5}})^{x+1}=\frac{125}{27}$

v) $4^{x-2}-2^{x+1}=0$

vi) $8.2^{2x}+4.2^{x+1}=1+2^x$

vii) $2^{2x}+2^{x+2}-4.2^3=0$

viii) $(\sqrt{3})^{x-3}=(\sqrt[4]{3})^{x+1}$

ix) If $5^{x+1}=25^{x-2}$; find the value of $3^{x-3} \,.\, 2^{3-x}$

x) $(81)^{3/4}-(\frac{1}{32})^{-2/5}+x(\frac{1}{2})^{-1} \,.\, 2^0=27$

Question 6 :

Prove that $(\frac{x^a}{x^b})^{a+b-c} . (\frac{x^b}{x^c})^{b+c-a} . (\frac{x^c}{x^a})^{c+a-b}=1$

Question 7:

If $a^x=b^y=c^z$ and $b^2=ac$, prove that $y=\frac{2xz}{x+z}$

Question 8:

If $m \neq n$ and $(m+n)^{-1}(m^{-1}+n^{-1})=m^x n^y$; show that $x+y+2=0$

Question 9:

Prove that : $\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}=\frac{2b^2}{b^2-a^2}$

Question 10:

If $3^{x+1} = 9^{x-3}$, find the value of 2^{1+x}.

THE END