

Girls' High School & College, Prayagraj

Session :2020-21

Class: 8 (A,B,C,D,E)

Subject : Mathematics

Worksheet : 04

INSTRUCTIONS : Parents kindly ensure that the student understands the given examples to solve the questions that follow. Students can also refer to class 7 or 8 Maths book or internet.

Chapter : EXPONENTS

Exponent- If a is a real number n is an integer, we know

$a \times a \times a \times a \times a \times \dots \times a$ n times = a^n , where a^n is called an exponential expression with base a and exponent (or index or power) n .

a^n is read as **a raised to the power n .**

LAWS OF EXPONENTS

1) Product law : $a^m \times a^n = a^{m+n}$

e.g. : $3^5 \times 3^4 = 3^{5+4} = 3^9$

2) Quotient law : $\frac{a^m}{a^n} = a^{m-n}$

e.g. : $\frac{2^{12}}{2^7} = 2^{12-7} = 2^5$

3) Power law : $(a^m)^n = a^{mn}$

e.g. : $(3^5)^2 = 3^{5 \times 2} = 3^{10}$

$(7^{-2})^3 = 7^{-2 \times 3} = 7^{-6}$

NOTE: i) if n is even, $(-a)^n$ is positive.

ii) if n is odd, $(-a)^n$ is negative.

$$\text{e.g. } (-2)^3 = -2 \times -2 \times -2 = -8$$

$$(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$$

$$(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$$

4) For any non zero rational number a

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad a^n = \frac{1}{a^{-n}}$$

i.e. a^{-n} and a^n are reciprocal of each other

$$\text{e.g.:} \quad \text{i) } 5^{-3} = \frac{1}{5^3}$$

$$\text{ii) } 7^3 = \frac{1}{7^{-3}}$$

SOLVE THE FOLLOWING QUESTIONS ACCORDING TO THE EXAMPLES GIVEN

EXAMPLE 1

$$\text{Evaluate: } (2^{-1} \div 5^{-1})^2 \times \left(\frac{-5}{8}\right)^{-1}$$

$$\text{Solution : } (2^{-1} \div 5^{-1})^2 \times \left(\frac{-5}{8}\right)^{-1}$$

$$= \left(\frac{1}{2} \div \frac{1}{5}\right)^2 \times \left(\frac{8}{-5}\right)$$

$$= \left(\frac{1}{2} \times \frac{5}{1}\right)^2 \times \frac{8}{-5}$$

$$= \frac{25}{4} \times \frac{8}{-5}$$

$$= \frac{5 \times 2}{-1}$$

$$= -10 \text{ answer}$$

Question 1: Evaluate:

$$\text{i)} (3^{-1} \times 9^{-1}) \div 3^{-2}$$

$$\text{ii)} (3^{-1} \div 4^{-1})^2$$

$$\text{iii)} (2^2 + 3^2) \times \left(\frac{1}{2}\right)^2$$

EXAMPLE 2

$$\text{Evaluate : } \left\{\left(\frac{-3}{2}\right)^{-3}\right\}^2$$

$$\text{Solution : } \left\{\left(\frac{-3}{2}\right)^{-3}\right\}^2$$

$$= \left(\frac{-3}{2}\right)^{-3 \times 2}$$

$$= \left(\frac{-3}{2}\right)^{-6}$$

$$= \left(\frac{-2}{3}\right)^6$$

$$= \frac{64}{729} \text{ answer}$$

Question 2: Evaluate:

$$\text{i)} \left[\left(\frac{-3}{4}\right)^{-2}\right]^2$$

$$\text{ii)} \left\{\left(\frac{3}{5}\right)^{-2}\right\}^{-2}$$

$$\text{iii)} (5^{-1} \times 3^{-1}) \div 6^{-1}$$

EXAMPLE 3

If $3^{3x-1} \div 9 = 27$, find the value of x

$$\text{Solution : } 3^{3x-1} \div 9 = 27$$

$$3^{3x-1} \times \frac{1}{9} = 27$$

$$3^{3x-1} \times \frac{1}{3^2} = 3^3$$

$$3^{3x-1-2} = 3^3$$

$$3^{3x-3} = 3^3$$

i.e. $3x - 3 = 3$

$$3x = 6$$

$$x = 2$$

Question 3: Solve the following:

i) Find x , if $9 \times 3^x = (27)^{2x-3}$

ii) If $1125 = 3^m \times 5^n$; find m and n . (Hint : find prime factors of 1125)

MORE ABOUT EXPONENTS

1) $(a \times b)^n = a^n \times b^n$

2) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

3) $a^0 = 1$; if $a \neq 0$

4) $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$; $a \neq 0$

5) $\sqrt[n]{a} = a^{1/n}$ and $\sqrt[n]{a^m} = a^{m/n}$

EXAMPLE 4

Evaluate: $4^{\frac{3}{2}} \times (125)^{\frac{-2}{3}}$

Solution: $4^{\frac{3}{2}} \times (125)^{\frac{-2}{3}} = 2^{2 \times \frac{3}{2}} \times 5^{3 \times \frac{-2}{3}}$

$$= 2^3 \times 5^{-2}$$

$$= 8 \times \frac{1}{5^2}$$

$$= \frac{8}{25} \text{ answer}$$

EXAMPLE 5

Evaluate : $(\frac{8}{27})^{2/3} \div (32)^{-2/5}$

Solution : $(\frac{8}{27})^{2/3} \div (32)^{-2/5} = (\frac{2}{3})^{3 \times 2/3} \div 2^{5 \times -2/5}$

$$= \frac{4}{9} \times \frac{1}{2^{-2}}$$

$$= \frac{4}{9} \times 2^2$$

$$= \frac{4 \times 4}{9}$$

$$= \frac{16}{9}$$

$$= 1\frac{7}{9} \text{ answer}$$

Question 4: Evaluate:

i) $1^8 \times 3^0 \times 5^3 \times 2^2$

ii) $(4^7)^2 \times (4^{-3})^4$

iii) $(\frac{2}{3})^{-4} \times (\frac{27}{8})^{-2}$

iv) $(243)^{2/5} \div (32)^{-2/5}$

v) $(8+4+2)^0$

vi) $(125a^{-3})^{1/3}$

vii) $(625)^{-3/4}$

EXAMPLE 6

Simplify : $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$

Solution : $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$
 $= x^{(a-b)(a+b)} x^{(b-c)(b+c)} x^{(c-a)(c+a)}$
 $= x^{a^2-b^2+b^2-c^2+c^2-a^2}$
 $= x^0$
 $= 1$

Question 5: Solve the following:

i) Show that $\left(\frac{x^a}{x^b}\right)^{a-b} \cdot \left(\frac{x^b}{x^c}\right)^{b-c} \cdot \left(\frac{x^c}{x^a}\right)^{c-a} = 1$

ii) Prove that $(m+n)^{-1} (m^{-1} + n^{-1}) = (mn)^{-1}$

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