GIRLS' HIGH SCHOOL AND COLLEGE

2020 – 2021

CLASS -11 A&B

PHYSICS

WORKSHEET- 01

Chapter- UNITS & MEASUREMENT

Topic – ERROR ANALYSIS

INSTRUCTIONS: Parents kindly instruct you ward to visit the websites https://physicsabout.com

https://www.animations.physics.unsw.edu.au

https://www.wikipedia.org

https://physics.unc.edu

or any other relevant site

or refer Nootan ISC 11 Physics-11 by Kumar & Mittal (Nageen Prakashan) or Physics -11 by DK Tyagi(Balaji Publications) to answer the following questions on the given topic.

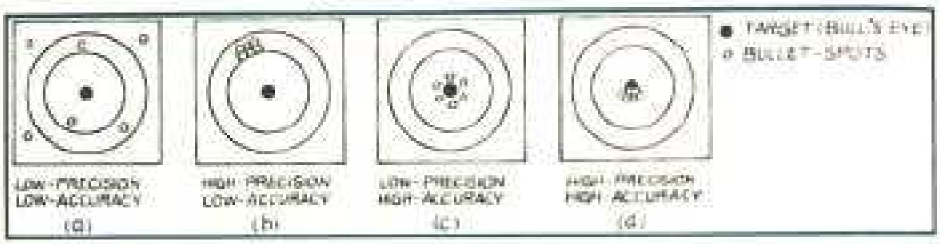
NOTE: The child should go through the subject matter thoroughly before answering the questions that follow. Students you should also go through your class 9^{th} & 10^{th} books for significant figures and order of magnitude concept. Go through the following information first-

(v) Electric Oscillator : In India, the domestic supply of a.c. is done at frequency of 50 Hz. The correspondin time period $\left(T = \frac{1}{\text{frequency}}\right)$ works out to 0.028. The rotation of a synchronous electric motor (run b this a.c.) can be used to develop a time scale for measurement of small time intervals.

(vi) Electronic Oscillator : Such oscillators are designed using semiconductor devices and produ electromagnetic waves of very high frequencies. These oscillations can be utilized for measurement extremely low time intervals.

Accuracy and Precision

In common language the phrases accuracy and precision convey almost the same meaning but in scienc same is not true; an instrument may give precise measurement but it can be quite inaccurate at the sam time. To understand the distinction between these two very closely related terms let us take an exampl in which a person is doing shooting practice and is allowed to shoot six bullets aiming at 'Bull's eye' a target frame consisting of circular regions of varying radius (Fig. 9 a, b, c and d).



(Fig. 9)

In Fig. 9 a , the shots hit the target frame in the outer circle and are scattered widely with respect t each other. Hence it is a case of poor precision and poor accuracy. In Fig. 9 b, the bullet marks are i the outer circular region away from the target but are clustered in a very small region. Thus, it is a cas of poor accuracy and high precision. Further in Fig. 9 c, although the bullet spots are scattered widel with respect to each other, yet these spots are in the internal circular region, very close to the Bull eye (the target) therefore, it is a case of poor precision and high accuracy. And finally in Fig. 9 d, th spots are clustered with respect to each other and simultaneously these are very close to the 'Bull's eye Therefore, it is a case of high precision and high accuracy.

To sum up it can be said, that *"the extent upto which an observed value agrees with the true value of quantity is known as its accuracy"*

And the instrument which gives repeated readings close to the true value of the physical quantity unde consideration, is an accurate instrument. On the other hand the precision of an observed value tells what resolution the quantity is measured. Thus, the measured values which are very close to each othe (may or may not be close to the true value) are precise values and an instrument which gives a tigh cluster of repeated results is a precise instrument.

More about Accuracy and Precision

The accuracy of an instrument depends more on systematic errors (for example calibration error, zer error etc.) present in it rather than some other factors. Therefore, it can be improved by re-calibratio or by applying proper corrections. Higher the accuracy, smaller is the error.

Thus, an error gives the indication of accuracy. On the other hand precision depends on the random errors. Therefore, it can not be eradicated. Moreover, higher the precision, larger is the number of significant figures. Thus, precision gives the indication of number of significant figures in a measurement. For example, let the true length of a rod is 5.346 cm. An experimenter 'A' measures the length of the rod using a metre scale of least count (resolution) 0.1 cm and measures the same as 5.3 cm. Further, another experimenter 'B' measures the length of the same rod with the help of a vernier callipers of least count 0.01 cm and records it as 5.24 cm.

The measurement of experimenter 'A' is more accurate for being very close to the true value (5.346 cm) but at the same time it is less precise as it is measured with an instrument of less resolution. On the other hand the length of the rod recorded by experimenter 'B' is less accurate but more precise at the same time, for having more number of significant figures.

Errors in Measurement

In the measurements taken during an experiment in the laboratory, two types of errors are present: error of observation and error present in the instrument. No measurement is perfectly accurate, whatsoever precise the instrument may be and whatever precisely the observation is taken. Uncertainty of measurement is technically called as 'error'. Error is always expressed in percentage.

To determine a physical quantity, we have to measure various quantities, which are related with that physical quantity by a formula. For example, to determine the density (ρ) of a metal block, we have to measure its mass (m) and its volume (V) which are related to ρ by a formula $\rho = m / V$. The accuracy in the value of ρ depends upon the accuracy of measurements of m and V. Measurement of these quantities involve errors which are of two types :

(a) Systematic errors, and (b) Random errors.

(a) Systematic Errors : These errors occur constantly in an experiment repeated under identical conditions. These errors come into existence by virtue of a definite rule. Therefore, once the rule which governs these errors, is identified these errors can be eradicated by applying proper corrections to the result obtained. These can be further subclassified as follows :

(i) Instrumental Errors : These errors are due to shortcoming in the instrument like error due to defective alignment of the instrument, zero error in a vernier callipers, screw gauge and spherometer, backlash error etc.

It should be noted here that sometimes, wrong choice specification of an instrument becomes the cause of an error in a measurement.

For example, a low resistance voltmeter yields a wrong result if connected across a load (a high resistance).

(ii) Error Due to Imperfection : This error occurs due to some imperfection of the apparatus used in an experiment. For example, in heat experiments the loss of heat due to radiation is an unwanted phenomenon. However, good the insulation may be, some heat is likely to be lost by radiation.

(iii) Gross Errors : These are the errors introduced due to casual approach of the experimenter, such as improper adjustment of the experimental setup in observation part and computational mistakes etc. in deduction part. Moreover, these errors can be avoided by careful working.

(b) Random Errors : These arise due to (i) small changes in the conditions of the experiment and (ii) incorrect judgement of the observer in taking readings.

Examples : (i) In finding the null point in the metre bridge or the potentiometer experiment, if we take the readings of the null point three or four times, we notice that there are small variations in the readings. This is an example of random error. It may arise due to change in conditions of the experiment, such as the heating effect of current or fall in e.m.f. of the cell.

(ii) Suppose we are measuring the weight of a body by a spring balance. Then there may be an error in assessing the correct position of the pointer on the scale; it may lie between two consecutive markings on the scale. Such an error due to incorrect judgement of the observer is also called random error.

The exact cause of random error cannot be traced.

Method of Minimising Random Errors : Random error can be minimised by taking a large number of readings of the same quantity. Then it is very likely that the majority of readings may have small errors which might be positive or negative. The error will be positive or negative depending upon whether the observed reading is above or below the correct value. Thus, ***random error can be minimised by taking***

the arithmetic mean of a large number of readings of the same quantity. The mean will be very close to the correct value. If one or two observations differ widely from the rest, these should be rejected while taking the mean.

Method of Expressing the Error : The error estimated in the measurement of a physical quantity can be expressed in following ways :

(a) Absolute error

(b) Mean absolute error (or mean error)

(c) Fractional or relative error

(d) Percentage error.

The methods of calculating the error in above ways are described below.

(a) Absolute Error : The difference between the true value and the measured value of the physical quantity is termed as the absolute error in the measurement, *i.e.*,

$$\Delta a = a - a_{\text{measured}},$$

where a represents the true value of the measured quantity. The absolute error has the same unit as that of measured physical quantity.

(i) When true value is not given and a single measured value is given, then absolute error in measured quantity a is given as

$$\Delta a = \text{least count of measuring instrument.}$$

For example, if the measured value of length of a pencil is given as $l = 10.8$ cm, then absolute error in measured length

$$\Delta l = 0.1 \text{ cm.}$$

(ii) When true value is not given, but a number of measured values $a_1, a_2, a_3, \ldots, a_n$ are given, then the true value a is taken as the mean of measured values, *i.e.*,

$$a = a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

The absolute error in each measurement is then determined as

$$\Delta a_1 = a_{\text{mean}} - a_1$$
$$\Delta a_2 = a_{\text{mean}} - a_2$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$\Delta a_n = a_{\text{mean}} - a_n$$

(b) Mean Absolute Error or Mean Error : The arithmetic mean of absolute errors of the different measurements is called as mean error.

$$\text{Mean error, } \overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n}$$

or

$$\overline{\Delta a} = \frac{1}{n} \sum_{i=1}^{i=n} |\Delta a_i|$$

It should be noted that while calculating mean error, the errors in different observations should be considered with positive sign only, whether the error is positive or negative.

(c) Relative Error or Fractional Error : The ratio of the mean absolute error to the true value of the measured quantity is known as relative or fractional error.

Thus,

$$\text{fractional error} = \frac{\overline{\Delta a}}{a}$$

(d) Percentage Error : If the fractional error is multiplied by 100, then we get what is known as the percentage error.

Thus,

$$\text{percentage error} = \frac{\overline{\Delta a}}{a} \times 100$$

An Important Note : If a is the true value and $\overline{\Delta a}$ is the mean error then the magnitude of the quantity may lie between $a + \overline{\Delta a}$, and $a - \overline{\Delta a}$. Thus, the result is expressed as $a \pm \overline{\Delta a}$. In this $\overline{\Delta a}$ is also known as limit of error. For example, if the length of the object is recorded as, 3.02, 3.03, 3.01, 3.03 and 3.01cm then we can express out the result as, 3.02 ± 0.01 cm. Therefore, the result has an error swing of ± 0.01 cm or percentage error in the measurement is $\frac{0.01}{3.02} \times 100 = 0.33\%$.

Least Count of Measuring Instruments : Accuracy in Measurement

The quality of a measuring instrument is recognised by its least count. The least count of a measuring instrument is the smallest quantity which it is capable of measuring. For example, the least count of an ordinary scale is 0.1 cm, that of a vernier callipers is 0.01 cm and that of a screw gauge and a spherometer is 0.001 cm. A good instrument should have as small a least count as possible.

Accuracy : Inspite of elimination of random and systematic errors, every measurement has a certain limit of accuracy depending upon the least count of the measuring instrument. Suppose a length measured by a metre scale is 24.7 cm. The maximum uncertainty in this measurement may be ± 0.1 cm (which is the least count of the metre scale). It means that the error in the measurement is 0.1 in 24.7 or the fractional error is $\frac{0.1}{24.7}$; or the percentage error is $\frac{0.1}{24.7} \times 100 = \frac{100}{247} = 0.4\%$.

If Δx be the error in a measurement x, then

$$\text{percentage error} = \frac{\Delta x}{x} \times 100.$$

Suppose, in an experiment, a physical quantity S is determined by measuring three quantities x, y and z, which are measured by different instruments. S may be related with x, y and z in the following manner:

$$S \propto (x)^a (y)^b (z)^{-c}$$

or

$$S = k (x)^a (y)^b (z)^{-c}$$

where k is a constant and a, b and c are powers. Let the expected small errors in the measurement of the quantities x, y and z be respectively $\pm \delta x$, $\pm \delta y$ and $\pm \delta z$, so that the error in S by using these measured quantities is $\pm \delta S$. The numerical values of δx, δy and δz are given by the least count of the instruments used to measure them. Now, to calculate the error in the value of S, we take logarithm of both sides of above equation. Then, we have

$$\log S = \log k + a \log x + b \log y - c \log z.$$

By partial differentiation, we have

$$\frac{\delta S}{S} = 0 + a\frac{\delta x}{x} + b\frac{\delta y}{y} - c\frac{\delta z}{z}.$$

Here δS represents the expected error in the value of S. The values δx, δy and δz may be positive or negative. It is possible that the signs of the errors be such that they affect the result in the same manner. In this condition, the fractional error $\delta S/S$ in the value of S will be *maximum*. For example, if in the above example δx and δy be of positive sign (+) and δz of negative sign (–), then in the above equation all the terms of right-hand-side will be added and the value of $\delta S/S$ will be maximum. Hence the maximum expected error in the value of S is

$$\left|\frac{\delta S}{S}\right|_{max} = a\frac{\delta x}{x} + b\frac{\delta y}{y} + c\frac{\delta z}{z}$$

The maximum expected percentage error in the value of S is

$$\left|\frac{\delta S}{S}\right|_{max} \times 100 = \left(a\frac{\delta x}{x} \times 100\right) + \left(b\frac{\delta y}{y} \times 100\right) + \left(c\frac{\delta z}{z} \times 100\right).$$

This is the maximum percentage error which may arise due to the limit of accuracy of the measuring instruments used in the experiment. This is called 'permissible error'. If the percentage error in the value of physical quantity determined by an experiment is greater than this, then it will be assumed either due to carelessness on the part of the student or due to some uncontrolled conditions.

19 Combination of Errors

When a number of quantities are involved in the final calculation, then errors associated with the measurement of all quantities will affect the end result. In this way the error in the final result depends upon :

(i) The error in the individual measurement.

(ii) The nature of the mathematical operation done on them to arrive at the final result.

We therefore, need rules for calculating the combined error associated with different mathematical operations; which are as under.

(a) Sum and Difference : Suppose limiting errors in two physical quantities x and y are, $\pm \Delta x$ and $\pm \Delta y$ respectively and,

$$z = x + y$$

Further let, limiting error in the sum z is $\pm \Delta z$

then, $$z \pm \Delta z = (x \pm \Delta x) + (y \pm \Delta y)$$

or $$\pm \Delta z = \pm \Delta x \pm \Delta y$$

Thus, the maximum possible error in z, is given by

$$\Delta z = \Delta x + \Delta y$$

Consider, next the difference.

Let, $$z = x - y$$

or $$z \pm \Delta z = (x \pm \Delta x) - (y \pm \Delta y)$$

therefore, $$\pm \Delta z = \pm \Delta x \mp \Delta y$$

The maximum possible error in z, $\Delta z = \Delta x + \Delta y$

Thus, when two quantities are added or subtracted the limiting error in the final result is the sum of limiting errors in the quantities involved.

(b) Product and Quotient : Let, $$z = xy$$

$$z \pm \Delta z = (x \pm \Delta x) \cdot (y \pm \Delta y)$$

$$= xy \pm x \cdot \Delta y \pm y \cdot \Delta x \pm \Delta x \cdot \Delta y$$

Since $\Delta x \cdot \Delta y$ is very small compared to the other terms and so it is neglected.

Hence, $$\pm \Delta z = \pm x \cdot \Delta y \pm y \cdot \Delta x$$

Dividing both sides by z $(= xy)$, we have,

$$\frac{\pm \Delta z}{z} = \pm \frac{\Delta y}{y} \pm \frac{\Delta x}{x}$$

Thus, the maximum possible error in z,

$$\left|\frac{\Delta z}{z}\right|_{max} = \frac{\Delta y}{y} + \frac{\Delta x}{x}$$

where, $\frac{\Delta y}{y}$ and $\frac{\Delta x}{x}$ are relative errors in y and x respectively.

And the percentage error,

$$\left|\frac{\Delta z}{z}\right|_{max} \times 100 = \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right) \times 100$$

Consider next the quotient/division.

Let
$$z = \frac{x}{y}$$

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = (x \pm \Delta x)(y \pm \Delta y)^{-1}$$

or
$$z \pm \Delta z = x\left(1 \pm \frac{\Delta x}{x}\right) y^{-1}\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$$

Expanding with the help of 'Binomial theorem' and neglecting higher powers of $\frac{\Delta y}{y}$ we have ,

$$z \pm \Delta z = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \mp \frac{\Delta y}{y} \pm \ldots\right)$$

But, $z = \frac{x}{y}$,

Thus
$$1 \pm \frac{\Delta z}{z} = 1 \pm \frac{\Delta x}{x} \mp \frac{\Delta y}{y} \mp \frac{\Delta x \Delta y}{xy}$$

the last term is small compared to other terms, so it is neglected.
Thus, maximum possible error in z

$$\left|\frac{\Delta z}{z}\right|_{max} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

and, therefore, the maximum percentage error in z

$$\left|\frac{\Delta z}{z}\right|_{max} \times 100 = \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right) \times 100$$

Thus when two quantities are multiplied or divided, the fractional error in the final result is the sum of the fractional errors in the quantities to be multiplied or to be divided.

Alternative :

Let,
$$z = xy$$

taking logarithm on both sides,

$$\log z = \log x + \log y$$

On differentiating partially, we have,

$$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

and in quotient

Let
$$z = \frac{x}{y}$$

taking logarithm on both sides,

$$\log z = \log x - \log y$$

On differentiating we have, $\frac{\Delta z}{z} = \frac{\Delta x}{x} - \frac{\Delta y}{y}$

Therefore, the maximum possible error in z

$$\left|\frac{\Delta z}{z}\right|_{max} = \frac{\Delta x}{x} + \frac{\Delta y}{y}.$$

(c) Power of a Measured Quantity :

Let , $z = x^m$

taking logarithm on both sides,

$$\log z = m \log x$$

Differentiating $\frac{\Delta z}{z} = m\frac{\Delta x}{x}$

and the percentage error in z

$$\left(\frac{\Delta z}{z}\right) \times 100 = m\left(\frac{\Delta x}{x}\right) \times 100$$

After going through the above subject matter thoroughly you can now attempt the following questions :-

Q1) Define error.

Q2) Differentiate between accuracy and precision.

Q3) Name the different kinds of errors that can be incurred during measurement.

Q4) What are systematic errors? Give its types with proper examples.

Q5) How can random errors be reduced?

Q6) What is order of magnitude? Find order of magnitude of-

(i) 2.32×10^{12} and (ii) 4.56×10^{-12} .

Q7) The radius of hydrogen atom is about 0.5 Å. Find the total atomic volume (in m^3) of a mole of hydrogen atoms. What is order of magnitude of this volume?

Q8) A laser beam aimed at the moon takes 2.56 s to return after reflection at moon's surface. Find the radius of the lunar orbit around earth and give its order of magnitude.

Q9) While measuring the length of a wooden block 5 readings were taken which were as follows- 4.53 cm, 4.53 cm, 4.52 cm, 4.51 cm,

4.53 cm. What are the absolute errors in the readings? Also find the mean absolute error.

Q10) While measuring the thickness of a measuring cylinder using vernier the mean internal diameter was 5.25 cm and mean external diameter was 5.68 cm. What is the thickness of the cylinder?

Q11) What is fractional error and percentage error?

Q12) The thickness of a coin as found by a screw gauge was 0.259 cm. Find its fractional error.

Q13) Several observations made in an experiment of simple pendulum by a student for its time period were as follows- 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.82 s. Find (i) mean period of oscillation, (ii) absolute error in each reading , (iii) mean absolute error, (iv) fractional error and

(v) percentage error.

Q14) If percentage error in measuring the side of a cube is 3% then find percentage error in measurement of its volume.

Q15) The error in measurement of diameter of a sphere is 1%. What will be the percentage error in its calculated volume?

Q16) Given $Z = \frac{A^4 B^{1/3}}{C D^{3/2}}$, where A, B, C, D are measured quantities. What is the maximum fractional error in Z?

Q17) Two students measure the length of a rod as 2.5 m and 2.54 m. Which measurement is more accurate and why?

Q18) The measured lengths of two rods are recorded as (23.8 ± 0.2) cm and (15.3 ± 0.2) cm. Write the sum and difference of the two lengths with their correct error limits.

Q19) A physical quantity S is given by, S = $\frac{a^2b^3}{c\sqrt{d}}$. If errors of measurement in

a, b, c and d are 2%, 5%, 3% and 4% respectively, find the percentage error in value of S.

Q20) The centripetal force on a particle moving along circular path is given by F = $\frac{mv^2}{r}$. The mass (m), velocity (v), and radius (r) of circular path of an object are 0.5 kg, 10m/s and 0.4 m respectively, then what is the percentage error in the centripetal force?

END