GIRLS' HIGH SCHOOL AND COLLEGE

2020 – 2021

CLASS -12 A&B

PHYSICS

WORKSHEET- 04

Chapter- ELECTRIC POTENTIAL

Topic – ELECTRIC POTENTIAL

INSTRUCTIONS: Parents kindly instruct your ward to visit the websites https://www.physicsclassroom.com

https://www.scoreacademy.in

https://www.wikipedia.org

or any other relevant site

or refer Nootan ISC 12 Physics-12 by Kumar & Mittal (Nageen Prakashan) or Physics -12 by DK Tyagi(Balaji Publications) to answer the following questions on the given topic.

NOTE: Electric Potential is an important topic from the point of view of the ISC Examinations. So kindly go through this topic given below carefully.

Electric Potential

Syllabus

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges; equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.

INTRODUCTION

Along with electric field intensity, electric potential is the parameter used to map the effect of electric charges. In fact, being scalar, it is more simple and easily expressible and assessible parameter. In this chapter, the concept of potential and related topics have been discussed.

1 Electric Potential

The electric field produced by a charge can be represented in two ways : (i) by the intensity of electric field $\vec{E}$ at a point in the field and (ii) by the electric potential V. The electric field intensity $\vec{E}$ at a point in the electric field refers to force acting on a unit charge when placed at that point and is a vector quantity. On the other hand, the electric potential at a point is a scalar quantity and refers to the work done in carrying a unit charge from a reference point to that point. In defining the potential at a point in the electric field, the reference point is taken outside the field (infinity for this definition) where the potential is taken zero.

The work done by an external agent in carrying a unit positive test charge from infinity to a point in the electric field is called the electric potential at that point.

Thus, if W be the work done in carrying the test charge $+q_0$ from infinity to a point P in the electric field, then potential at P is given by

$$V = \frac{W}{q_0} \qquad \text{...(i)}$$

SI Unit of Potential : The unit of work W is 'joule' (J) and the unit of charge q_0 is 'coulomb' (C). Hence, according to Eq. (i), the SI unit of potential will be 'joule/coulomb'. This is called 'volt' (V). Thus,

$$1 \text{ volt} = 1 \text{ joule/coulomb}$$

or

$$1\text{ V} = 1\text{ J C}^{-1}.$$

Thus, ***if 1 joule of work is done in carrying a test charge of 1 coulomb from infinity to a point in an electric field, then the potential at that point will be 1 volt.***

$$\text{Dimensions of electric potential} = \frac{\text{dimensions of work}}{\text{dimensions of charge}} = \frac{[\text{M L}^2\text{ T}^{-2}]}{[\text{A T}]} = [\text{M L}^2\text{ T}^{-3}\text{A}^{-1}]$$

Physical Interpretation of Electric Potential : We know that a liquid always flows from a higher level to a lower level. Heat also flows from a body at a higher temperature to a body at a lower temperature. Similarly, positive charge always flows from higher potential to lower potential. Just as flow of liquid does not depend upon the quantity of liquid, but depends only upon the level of liquid; and flow of heat also does not depend upon the amount of heat; in a similar way, the flow of positive charge does not depend upon the quantity of charge.

Thus, the electric potential of a conductor is its electric state which determines the direction of flow of charge when the given conductor is connected to another conductor. ***Positive charge always flows from a higher potential to a lower potential, while negative charge always flows from a lower to a higher potential until the potentials become equal.***

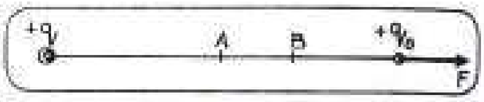
Note

(i) Electric potential for the flow of charge is analogous to liquid level for the flow of liquid or the temperature for the flow of heat. Just as liquid flows from higher level to lower level or heat flows from body at higher temperature to that at lower temperature, charge flows from body at higher potential to the body at lower potential.

(ii) For the potential earth is considered as the reference body. It means the electric potential of earth is taken zero and it is not affected no matter whatever large charge is given to it or taken from it.

(iii) At all points where electric field does not exist; electric potential is taken zero.

2 Potential Difference



(Fig. 1)

In definition of potential, the reference point (of zero potential) is taken at infinity (outside the field). If we take two points A and B in the electric field (Fig. 1) then potentials at A and B will be defined as above. The difference in these potentials is termed as 'potential difference' between the points A and B. Independently, it is defined as follows :

"The work done by an external agent in carrying a unit positive test charge from one point to the other point in an electric field is called the potential difference between those points."

Thus, if W be the work done in carrying the test charge q_0 from B to A, then the potential difference between A and B is given by

$$V_A - V_B = \frac{W}{q_0} \quad \text{...(ii)}$$

Like potential, *SI* unit of potential difference is volt and it is a scalar quantity. If, in taking a positive test charge from the point B to the point A, work is done by an external agent against the electric force, then the potential of A is said to be higher than the potential of B. (In Fig. 1, the potential of point A is higher than the potential of point B.) This also means that in an electric field a free positive charge moves from a region of higher potential to a region of lower potential. Conversely, a free negative charge moves from lower potential to higher potential.

If, in Fig. 1, the charge producing the electric field were $-q$, then in taking the positive test charge q_0 from B to A, work would have been done by the electric force itself. In that case, the potential of the point A would have been lower than the potential of the point B.

Note

(i) If a charge (say q_0) is taken from initial point A (potential V_A) to the point B (potential V_B). The work done by the external agent is

$$W = q_0 (V_{\text{final}} - V_{\text{initial}}) = q_0 (V_B - V_A).$$

In this q_0 is taken with proper sign (+ or −).

(ii) If initial point is outside the electric field, then

$$V_{\text{initial}} = 0.$$

(iii) If work W comes out to be negative, then work is obtained by external agent (*i.e.*, work is done by electric field itself).

3 Electron-Volt or eV

Electron-volt is a very small unit of work (or energy), which is used in atomic physics. 1 electron-volt is the work done in taking one electron from one point to the other, when the potential difference between these points is 1 volt. In other words, 1 electron-volt is the (kinetic) energy which an electron acquires when accelerated through a potential difference of 1 volt.

According to the definition of potential difference, if the potential difference between two points A and B is $V_A - V_B$, then the work done in taking a test-charge q_0 from the point B to the point A is

$$W = q_0 \, (V_A - V_B)$$

If $q_0 = e = 1.6 \times 10^{-19}$ coulomb, $V_A - V_B = 1$ volt, then $W = 1$ electron-volt. Thus,

$$1 \text{ electron-volt} = 1.6 \times 10^{-19} \text{ coulomb} \times 1 \text{ volt}$$

$$\mathbf{1\ electron\text{-}volt = 1.6 \times 10^{-19}\ joule.}$$

or

4 Electric Potential due to a Point-Charge

(Fig. 2)

Suppose a point charge of $+q$ coulomb is situated at a point O in a medium of dielectric constant K. Let P be a point, distant r from O, at which the electric potential is to be determined. For this, we must calculate the work done in bringing a test charge from infinity to P. Suppose, a test charge $+q_0$ is placed at point A, distant x from O, and away from P. By Coulomb's law, the magnitude of the electric force $\vec{F}$ acting on q_0 is given by

$$F = \frac{1}{4\pi\varepsilon_0 K} \frac{q\,q_0}{x^2} \text{ newton.} \qquad \ldots\text{(i)}$$

The direction of $\vec{F}$ is away from O. If the test charge $+q_0$ is displaced by dx to the point B, the work done

$$dW = F \cdot (-dx)$$

Negative sign is due to the reason that displacement dx is opposite to the direction of force. Hence work done in bringing $+q_0$ from infinity $(x = \infty)$ to the point P $(x = r)$ is

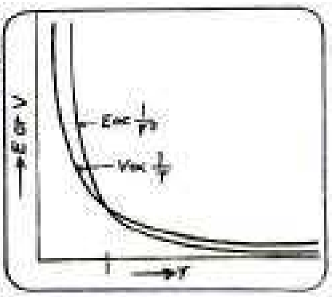
$$W = -\int_{\infty}^{r} F\,dx = -\frac{q\,q_0}{4\pi\varepsilon_0 K}\int_{\infty}^{r}\frac{1}{x^2}\,dx \qquad \text{[using (i)]}$$

$$= -\frac{q\,q_0}{4\pi\varepsilon_0 K}\left[-\frac{1}{x}\right]_{\infty}^{r} = -\frac{q\,q_0}{4\pi\varepsilon_0 K}\left(-\frac{1}{r}+\frac{1}{\infty}\right)$$

$$W = \frac{1}{4\pi\varepsilon_0 K}\cdot\frac{q\,q_0}{r}$$

Thus, the potential at P

$$V = \frac{W}{q_0} = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{r}.$$



(Fig. 3)

The figure (3) shows how the electric potential $V\left(\propto \frac{1}{r}\right)$ and the electric field $E\left(\propto \frac{1}{r^2}\right)$ varies with r.

For vacuum (or air) $K = 1$

$$\therefore \quad \mathbf{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\ Volt,}$$

where, $1/4\pi\varepsilon_0 = 9.0 \times 10^9$ newton-metre^2/coulomb^2 (Nm^2 C^{-2}).

Similarly, the potential at P due to a charge $-q$ is

$$V = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\text{ volt.}$$

Note

(i) In the above formula, the value of source charge q is put with sign. Thus, potential due to positive source is positive while due to the negative source charge is negative.

(ii) Positive value of potential at a point in the electric field means, work will have to be done by external agent in bringing unit positive charge from infinity to that point. In case of negative potential the above work is done by the field itself.

(iii) The potential of an electric field is its scalar (the energy) characteristic, whereas intensity is its vector (force) characteristic. Moreover, potential is a property of a point in the electric field therefore, it may be treated as a microscopic quanity.

(iv) Potential at a point in an electric field is arbitrary, depends on the choice of zero of reference. It is the potential difference between any two points that matters and not the absolute potential.

5 Potential due to a Group of Point Charges

Potential is a scalar quantity. Therefore, the potential at any point due to a group of point-charges is found by calculating the potential due to each charge (as if the other charges were not present), and then adding algebraically the quantities so obtained.

Thus, if a point is at distances r_1, r_2, r_3 and r_4 metre from the point charges $+q_1$, $+q_2$, $-q_3$ and $-q_4$ coulomb respectively, then the resultant potential at that point will be

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} - \frac{q_3}{r_3} - \frac{q_4}{r_4}\right]\text{ volt.}$$

If there are n point charges, the potential due to them at a point P will be

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

where r_i is the distance of the point P from the charge q_i.

If the charge distribution be continuous, then the summation in the above expression will be replaced by integration, that is,

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

where dq is a differential element of the charge distribution and r is its distance from the point at which V is to be calculated.

If the charge is spread continuously over an area A, then $dq = \sigma\, dA$, where σ is surface density of charge. Then, we have

$$V = \frac{1}{4\pi\varepsilon_0}\int_A\frac{\sigma\, dA}{r}$$

where $\int_A$ is a surface integral.

Similarly, if the charge is distributed continuously within a volume V, then

$$V = \frac{1}{4\pi\varepsilon_0}\int_V\frac{\rho\, dV}{r}$$

where r is volume density of charge and $\int_V$ is a volume integral.

6 Potential Gradient

We have seen that in an electric field, the value of potential changes from point to point depending upon the distance of the point from the source charge or the system of source charges. **The rate of change of potential with distance in the electric field is called the 'potential gradient'.**

Let dV is the change in potential with change in distance dr, then potential gradient is given by

$$\text{gradient } V = \frac{dV}{dr}$$

The unit of potential gradient is volt per metre (Vm^{-1}). It is a vector quantity whose direction is opposite to the direction of electric intensity.

Note

(i) Remember that the gradient of a scalar field (potential, temperature, etc.) is always a vector.

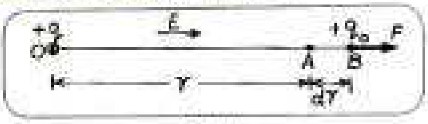
(ii) Whenever a parameter changes with distance, then the rate of change is called 'gradient'.

(iii) When $V = f(x, y, z)$ i.e., potential varies along all the three axes, then

$$\text{gradient } V = \hat{\mathbf{i}}\frac{\partial V}{\partial x} + \hat{\mathbf{j}}\frac{\partial V}{\partial y} + \hat{\mathbf{k}}\frac{\partial V}{\partial z}$$

where ∂ represents partial derivative and $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are unit vectors along x, y and z-axes.

7 Electric Field as Gradient of Electric Potential : Relation between E and V



(Fig. 4)

Let us consider the electric field E along the X-axis due to a point charge $+q$ at a point O (Fig. 4). Suppose A and B are two points distant r and $r + dr$ from O, where dr is vanishingly small. Let V and $V - dV$ be electric potentials at A and B respectively.

Suppose a small positive test charge q_0 is moved in the electric field from point B to point A. A force F acts on q_0 in the direction of the field, where

$$F = q_0 E. \qquad \text{... (i)}$$

Therefore, in moving q_0 from B to A, an external agent will have to work against the force F. If this work be dW, then

$$dW = F(-dr),$$

where $(-dr)$ is the magnitude of the displacement from B to A. Substituting the value of F from Eq. (i), we get

$$dW = -q_0 E\,dr$$

or

$$\frac{dW}{q_0} = -E\,dr. \qquad \text{... (ii)}$$

But by the definition of potential difference,

we have

$$\frac{dW}{q_0} = V - (V - dV) = dV. \qquad \text{... (iii)}$$

Comparing Eqs. (ii) and (iii), we get $\quad -E\,dr = dV$

or

$$\mathbf{E = -\frac{dV}{dr}}.$$

The quantity dV/dr is the rate of change of potential with distance and is known as 'potential gradient'. Thus, ***the electric field intensity at a point in an electric field in a given direction is equal to the negative potential gradient in that direction.*** The negative sign signifies that the potential decreases in the direction of electric field.

It is evident from the above relation that the electric field E can also be expressed in 'volt/metre' (V m

Thus,

$$1\ \text{N C}^{-1} = 1\ \text{V m}^{-1}.$$

In Fig. 5 are shown two metallic plates having positive and negative charges. If the plates are long and the distance between them is small, then the electric field produced between them will be uniform and directed from the positive plate to the negative plate. If the potentials of the plates be V_1 and V_2 volt and the distance between them be d metre, then the electric field E between the plates will be given by

$$E = \frac{V_1 - V_2}{d}\ \text{volt/metre.}$$

(Fig. 5)

Importance of the Relation between Electric Field and Electric Potential : This relation enables us to calculate the electric field intensity (a vector) at a point if potential (a scalar) at that point is known. For example, the electric potential due to a point charge q at a distance r is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

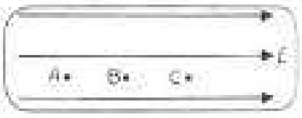
By symmetry, the electric field $\vec{E}$ must be directed radially outwards from a (positive) charge. Its magnitude equals the negative potential gradient in the radial direction. Thus,

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

Note

Potential decreases in the direction of electric intensity.
For the field shown

$$V_A > V_B > V_C$$

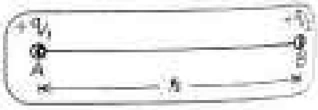


8 Electric Potential Energy of a System of Charges

Two (or more) electric charges attract or repel each other. Hence work is done in taking the charges away from each other, or in bringing near each other. This work is stored in the form of potential energy in the system of those charges. This is called 'electric potential energy' of the system. It can be defined in the following way :

The electric potential energy of a system of charges is the work that has been done in bringing those charges from infinity to near each other to form the system.

Suppose, a system AB is formed by two charges of $+q_1$ and $+q_2$ coulomb, placed in vacuum (or air) at a distance of r metre from each other (Fig. 6).



(Fig. 6)

To determine the electric potential energy of this system, suppose that the charge $+q_2$ is initially at infinity. Now, the electric potential at B due to the charge $+q_1$ is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}\ \text{volt.}$$

According to the definition of electric potential, the work done (= charge × potential) in bringing the charge q_2 from infinity to the point B is

$$W = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}\ \text{joule.}$$

This work is the electric potential energy U of the system $(q_1 + q_2)$. Hence,

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \text{ joule.} \qquad \ldots(i)$$

Note:

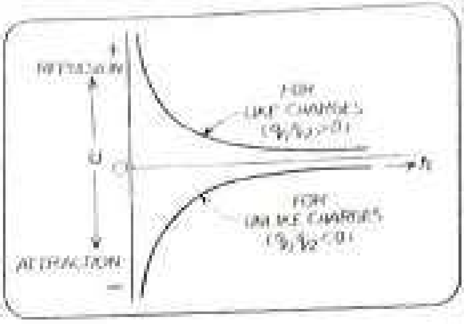
(i) In the above formula values of charges q_1 and q_2 are put with sign. Thus, both like charges, potential energy is positive while for unlike charges potential energy is negative.

(ii) Positive value of potential energy indicates net repulsion while negative value indicates net attraction.

(iii) Variation of U with distance r between the charges is shown in the figure for both $q_1 q_2 > 0$ and $q_1 q_2 < 0$.

(iv) Net attraction or net repulsion force between the charges can be obtained from potential energy function using the relation

$$F = -\frac{dU}{dr}$$



(v) For a system potential energy is minimum in the condition of stable equilibrium of the system (every system in the universe has the tendency to acquire minimum potential energy so that it may attain stable equilibrium).

(vi) When the system has more than two charges (say $n > 2$) then form the pairs (number of pairs $= \frac{n(n-1)}{2}$). Determine the potential energy of each pair using formula (i) and then add them algebraically. For example, if three charges q_1, $-q_2$ and q_3 be placed at the three corners of a triangle (figure), then the electric potential energy of the system will be given by

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1(-q_2)}{r_{12}} + \frac{(-q_2)(q_3)}{r_{23}} + \frac{q_1 q_3}{r_{13}}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[-\frac{q_1 q_2}{r_{12}} - \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}\right]$$

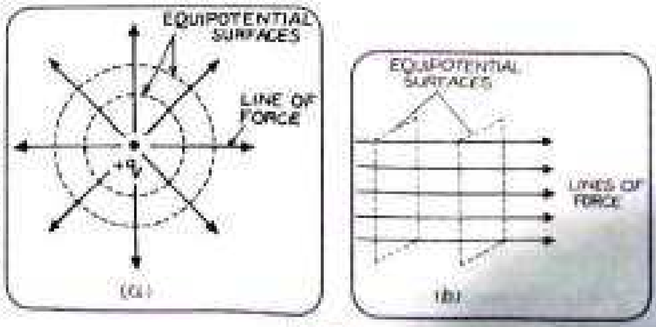
9 Equipotential Surfaces

Any surface over which the electric potential is same everywhere is called an equipotential surface.

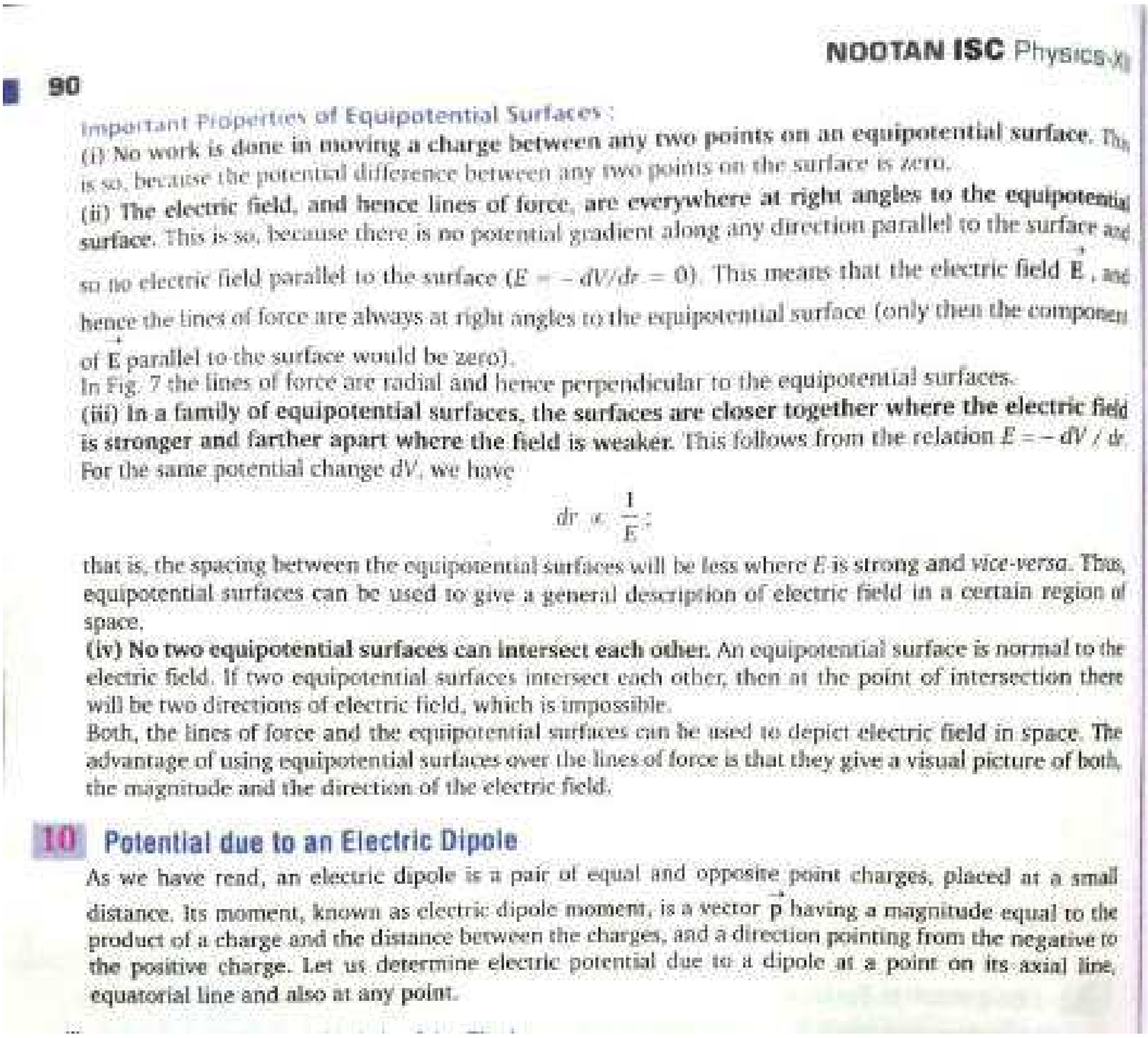
An equipotential surface may be the surface of a charged body or simply a surface in space. For example, the surface of a conductor is an equipotential surface. Equipotential surfaces can be drawn through a space in which there is an electric field. As an example, let us consider the electric field of an isolated point charge $+q$. The potential at a distance r from the charge is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}.$$

A sphere of radius r with centre at $+q$ is, therefore, an equipotential surface of potential $q/4\pi\epsilon_0 r$. In fact, all spheres centered on $+q$ are equipotential surfaces, whose potentials are inversely proportional to r (Fig. 7a). For a 'uniform' electric field, equipotential surfaces are a family of planes perpendicular to the lines of force (Fig. 7b).



(Fig. 7)



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Important Properties of Equipotential Surfaces :

(i) No work is done in moving a charge between any two points on an equipotential surface. Th is so, because the potential difference between any two points on the surface is zero.

(ii) The electric field, and hence lines of force, are everywhere at right angles to the equipotentia surface. This is so, because there is no potential gradient along any direction parallel to the surface an so no electric field parallel to the surface ($E = -dV/dr = 0$). This means that the electric field $\vec{E}$, an hence the lines of force are always at right angles to the equipotential surface (only then the componen of $\vec{E}$ parallel to the surface would be zero).

In Fig. 7 the lines of force are radial and hence perpendicular to the equipotential surfaces.

(iii) In a family of equipotential surfaces, the surfaces are closer together where the electric field is stronger and farther apart where the field is weaker. This follows from the relation $E = -dV/dr$. For the same potential change dV, we have

$$dr \propto \frac{1}{E};$$

that is, the spacing between the equipotential surfaces will be less where E is strong and *vice-versa*. Thus, equipotential surfaces can be used to give a general description of electric field in a certain region of space.

(iv) No two equipotential surfaces can intersect each other. An equipotential surface is normal to the electric field. If two equipotential surfaces intersect each other, then at the point of intersection there will be two directions of electric field, which is impossible.

Both, the lines of force and the equipotential surfaces can be used to depict electric field in space. The advantage of using equipotential surfaces over the lines of force is that they give a visual picture of both, the magnitude and the direction of the electric field.

10 Potential due to an Electric Dipole

As we have read, an electric dipole is a pair of equal and opposite point charges, placed at a small distance. Its moment, known as electric dipole moment, is a vector $\vec{p}$ having a magnitude equal to the product of a charge and the distance between the charges, and a direction pointing from the negative to the positive charge. Let us determine electric potential due to a dipole at a point on its axial line, equatorial line and also at any point.

Now answer the following questions: -

Q1 Define Electric potential difference. Give its mathematical expression.

Q2) Using the expression for potential difference define and obtain the expression for potential at a point. Give its unit and dimensional formula.

Q3) 16 J of work has to be done against an existing electric field to take charge of 0.02 C from A to B. How much is the potential difference between B and A?

Q4) A charge of 8 μC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-19} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0) via a point R (0, 6 cm, 9 cm).

Q5) What is eV? How much work is 1eV?

Q6) An electron which may be regarded as a small body of mass 9×10^{-31} kg carrying a negative charge o 1.6×10^{-19} C starts from a point of one conductor and reaches a second conductor with a velocity of 10^7 ms^{-1}. Calculate the potential difference in volt.

Q7) Using the definition of electric potential obtain he mathematical expression for Potential at a point due to a point charge.

Q8) What is the electric potential at the surface of platinum nucleus? The radius of the nucleus is 6.6×10^{-15} m and atomic number is 78.

Q9) Compare electric potential and electric field.

Q10) Give expression for potential due to a group of point charges for non-uniform and continuous distributions.

Q11) Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what points on the line joining the two charges is the electric potential zero?

Q12) Show that electric field at a point is also the negative potential gradient at that point. What is the significance of the negative sign?

Q13) The electric field at a point due to a point charge is 30 NC^{-1} and the electric potential at that point is 15 JC^{-1}. Calculate the distance of the point from the charge and magnitude of the charge.

Q14) Two points A and B are 3 m apart. A point charge $q = 2 \times 10^{-6}$ C is placed at 'O' at a distance 1 m from the point B on the line joining the two charges in between A and B. (i) Calculate potential difference between A and B.

(ii) What will be the result if the positions of A and B are interchanged?

(iii) What will be the result if the point B is located at 1 m distance from 'O' perpendicular to the line joining OA?

Q15) What is electric potential energy of a system of point charges? Obtain an expression for potential energy for a system of three charges scattered randomly in free space.

Q16) Three point charges -40 μC, 20 μC and 20 μC are placed at three corners of an equilateral triangle of side 10 cm. Calculate the potential energy of the system.

Q17) When is a surface called an equipotential surface? Give an example of such a surface.

Q18) Give the properties of equipotential surfaces.

Q19) State why – (i) Electric field is everywhere perpendicular to an equipotential surface,

(ii) In a family of equipotential surfaces, two surfaces are closer when field is stronger.

(iii) No two equipotential surfaces intersect.

Q20) A potential difference of 100 V is applied in between the two parallel

plates. Calculate the speed of a proton released from plate B, just before it hits plate A. Given $q = 1.6 \times 10^{-19}$ C and $m = 1.62 \times 10^{-27}$ kg.

END