GIRLS' HIGH SCHOOL AND COLLEGE, PRAYAGRAJ
Session 2020-21
CLASS–X (A,B,C,D,E,F)
SUBJECT–MATHEMATICS
WORKSHEET NO.-7

INSTRUCTIONS: – Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and websites given below.

NOTE – 1. Concise Mathematics ICSE Class X by R.K. Bansal

2. Understanding ICSE Mathematics Class X by M.L. Aggarwal

3. www.extramarks.com , www.topperlearning.com

Topic – Arithmetic Progresssion

INTRODUCTION– A group of numbers, which are arranged in a definite order following a certain rule, is called sequence.

Arithmetic Progression (A.P.): An arithmetic progression is a sequence (series) of numbers in which each term can be obtained by adding a certain quantity to its preceding term.

For example, the sequence 3, 8, 13, 18, 23-------- is an arithmetic progression in which every term (other than the first term) can be obtained by adding 5 to its preceding term i.e.

3+5=8, 8+5=13, 13+5=18 and so on.

In an A.P. the difference between two consecutive terms is called its **common difference** and is denoted by letter '**d**'.

Thus, if t_1, t_2, t_3, t_4,---------are consecutive terms (numbers) in A.P. ,its **first term** is t_1 and is denoted by **a**.

So, $a=t_1$

$d=t_2-t_1= t_3-t_2=t_4-t_3=$ ------- and so on.

e.g. 4, 6, 8, 10,-------

with first term a=4 and common difference d=6-4=2.

General Term of An Arithmetic Progression :

Let the first term of an A.P. be 'a' and its common difference be 'd',the terms of the A.P. can be taken as:

a, a+d, a+2d, a+3d,---------

First term = a=a+(1-1)d= a+(no. of term-1)d

Second term =a+d= a+ (2-1)d= a+(no. of term -1)d

Third term= a+2d= a +(3-1)d= a+(no. of term-1)d

On proceeding in the similar manner, we find:

n^{th} term of an A.P. = a+(n-1)d i.e. $t_n = a+(n-1)d$

Here, $t_n = a+(n-1)d$ is called the general term of the A.P. in which by putting n=1,2,3, etc.,we can get first term ,second term ,third term , respectively of the A.P. under consideration.

NOTE: If an A.P. has only n terms , its n^{th} term (last term) is denoted by letter **L**.

Thus for an A.P. with first term= a, common difference = d and number of terms= n , we have L=a+(n-1)d.

For an A.P., $t_n =a+(n-1)d$

=>if $d > 0$,the A.P. is increasing

if $d < 0$, the A.P. is decreasing and

if $d=0$, all the terms of the A.P. are same.

In general, if a sequence has n terms, then r^{th} term from its end

= $(n-r+1)^{th}$ term from the beginning

Sum of n terms of an A.P. :

Let for the given A.P. ,first term =a, common difference =d, number of terms =n and last term= L.

For an A.P. when

(i) a, n and L are known, take its sum **S= $\frac{n}{2}$ (a+L)**

(ii)a, n and d are known ,take **S= $\frac{n}{2}$ [2a+(n-1)d]**

NOTE: n^{th} term of an A.P. = Sum of n terms - Sum of (n-1) terms

e.g. 10^{th} term of A.P. = Sum of 10 terms – Sum of 9 terms

i.e. $t_{10} = S_{10} - S_9$

Three or more terms in A.P. :

1. When the sum of three consecutive terms of an A.P. is given ,we take the terms as: (a-d), a and (a+d).
2. When the sum of four consecutive terms of an A.P. is given, we take the terms as: (a-3d), (a-d), (a+d), (a+3d).
3. For consecutive five terms in A.P. : take terms as (a-2d), (a-d), a, (a+d) and (a+2d).

4. For consecutive six terms of A.P. :take terms as (a-5d), (a-3d), (a-d),(a+d), (a+3d) and (a+5d).

Arithmetic Mean :

If three numbers a, A and b are in arithmetic progression , then A is called **arithmetic mean** (A.M.) between a and b.

Since; a, A and b are in A.P.

=> A-a = b-A

=> 2A= a+b and A= $\frac{a+b}{2}$

∴ Arithmetic mean between a and b = $\frac{a+b}{2}$.

Properties of an A.P. :

Property 1: If same fixed non-zero number is added or subtracted from each term of an A.P., the resulting sequence is also an A.P.

e.g. (i) 5, 8, 11,14 ------are in A.P.

=> 5+7, 8+7, 11+7, 14+7,---are in A.P. [Adding 7 to each term]

and,5-7, 8-7, 11-7, 14-7,-----are also in A.P. [Subtracting 7 from each term]

Property 2: If each term of a given A.P. is multiplied or divided by a given non-zero fixed number, the resulting sequence is in A.P.

e.g. (i) 5, 8, 11, 14, -------are in A.P.

=> 5*8, 8*8 ,11*8, 14*8,-----are also in A.P. [Multiplying each term by 8]

(ii) 27, 25, 23, 21,-------- are also in A.P.

=> $\frac{27}{4}, \frac{25}{4}, \frac{23}{4}, \frac{21}{4}$-------are also in A.P. [Dividing each term by 4]

Example1: For an A.P. 7, 15 ,23, 31, ----------,write the first term, common difference and next two terms .

Solution :

First term of the given A.P. =7. Its common difference =15-7=8

Next two terms are 31+d= 31+8=39 and 39+8 =47

Example2: Find the A.P. whose second term is 12 and 7^{th} term exceeds the 4^{th} by 15.

Solution:

Second term =12 => a+d =12

7^{th} term – 4^{th} term =15 => (a+6d)-(a+3d)=15

i.e. 3d=15 => d=5 and a+d =12 => a=7

∴ The required A.P. = a, a+d, a+2d, a+3d, ----------

= 7, 7+5,7+2*5, 7+3*5,----------

=7 , 12, 17, 22,----------

Example3: Find the 12^{th} term from the end in A.P. 13, 18, 23, ----, 158.

Solution:

Let the given A.P. 13, 18, 23, -------, 158 has n terms

∴ 158 = 13+ (n-1)*5 [∵ t_n =a+(n-1)d]

=> n=30

12^{th} term from the end

= $(30 - 12 + 1)^{th}$ term from the beginning

= 19^{th} term from the beginning

= a+ 18d = 13+18*5 = 103

Example4: How many whole numbers, each divisible by 7, lie between 200 and 500?

Solution:

$\because \frac{200}{7} = 28\frac{4}{7}$ and $\frac{500}{7} = 71\frac{3}{7}$

The numbers between 200 and 500 and divisible by 7 are:

29*7, 30*7, 31*7,-------, 71*7

= 203, 210, 217,-------,497

It is an A.P. with first term a=203, common difference d=7

and last term L=497

L=a+(n-1)d => 497=203 +(n-1)*7

i.e. 294= (n-1)*7 => n-1= $\frac{294}{7}$ = 42 and n=43

$\therefore$ **There are 43 numbers between 200 and 500 which are divisible by 7 .**

Example5: For the A.P. : 10, 15, 20,------,195; find:

(i)the number of terms in the above A.P.

(ii)the sum of all its terms.

Solution:

Given: a=10, d=5 and last term L = 195

(i)If the given A.P. has n terms:

L = a+(n-1)d => 195= 10+(n-1)*5

=> 185= (n-1)*5

=> 37=(n-1) i.e. n=38

(ii) ∵ a=10, L=195 and n=38

∴ S = $\frac{n}{2}$[a+L]

= $\frac{38}{2}$ [10+195] = 19 *205=3895

Example6: How many terms of the A.P. 43, 39, 35, --------be taken so that their sum is 252?

Solution: Let the number of terms taken be n.

∵ a=43 and d= 39-43 = -4

∴ **S = $\frac{n}{2}$ [2a+ (n-1)d]** => 252 = $\frac{n}{2}$ [2*43+(n-1)*-4]

=> 252 *2 = n(86-4n+4)

=> 504=90n-4n^2

=> $2n^2$ – 45n +252=0

=> (n-12)(2n-21)= 0

=> n=12 or n= $\frac{21}{2}$

=> n=12 [∵ n≠ $\frac{21}{2}$]

∴ **The required number of terms = 12**

Example7: Insert two arithmetic means between 5 and 11.

Solution:

Let the required means (A.M.s) between 5 and 11 be A_1 and A_2

=> 5, A_1, A_2 and 11 are in A.P.

=> 11= 4^{th} term of this A.P.

=> 11= 5+3d i.e. d=2

$\therefore$ A_1 =5+d= 5+2=7 and A_2 =7+2=9

=> **Required A.M.s between 5 and 11 = 7 and 9 .**

Example8: In a school, students stand in rows. If 30 students stand in the first row, twenty-seven in the second row, twenty four in the third row and six in the last row; find how many rows are there and what is the total number of students?

Solution:

Sequence formed by the number of students standing in different rows = 30, 27, 24,----------,6

It is an A.P. with a=30, d= -3 and L=6

Since, a+(n-1)d = L

=> 30+(n-1)*-3 = 6 i.e. n=9

$\therefore$ **There are 9 rows**

Also, total number of students

=Sum of students in different rows

$= \frac{9}{2}$ [2*30 + (9-1)* -3] [$\because S_n = \frac{n}{2}$ {2a+(n-1)d}]

$= \frac{9}{2}$ [60-24] = $\frac{9}{2}$ * 36= 162

SOLVE THE FOLLOWING QUESTIONS:

Ques1- Find the 24^{th} term of the sequence :

12, 10, 8, 6,----------.

Ques2- If 5^{th} and 6^{th} terms of an A.P. are respectively 6 and 5.Find the 11^{th} term of the A.P.

Ques3-If t_n represents n^{th} term of an A.P., $t_2 + t_5 - t_3 = 10$ and $t_2 + t_9 = 17$, find its first term and its common difference.

Ques4- How many two-digit numbers are divisible by 3 ?

Ques5- Determine the A.P. whose 3^{rd} term is 16 and 7^{th} term exceeds the 5^{th} term by 12.

Ques6- Determine the value of k for which k^2+4k+8, $2k^2+3k+6$ and $3k^2+4k+4$ are in A.P.

Ques7-In an A.P., if m^{th} term is n and n^{th} term is m, show that its r^{th} term is (m+n-r).

Ques8-Find the sum of all natural numbers between 250 and 1000 which are divisible by 9.

Ques9- The first and the last terms of an A.P. are 34 and 700 respectively. If the common difference is 18, how many terms are there and what is their sum?

Ques10- Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.

Ques11-Insert three arithmetic means between 15 and 27.

Ques12- If the sum of first m terms of an A.P. be n and the sum of first n terms of the same A.P. be m, show that the sum of its (m+n) is –(m+n).

Ques13-Two cars start together in the same direction from the same place. The first car goes at uniform speed of $10kmh^{-1}$.The second car goes at a speed of $8kmh^{-1}$ in the first hour and thereafter increasing the speed by $0.5kmh^{-1}$ each succeeding hour. After how many hours will the two cars meet?

Ques14- A manufacturer of TV sets produces 600 units in the third year and 700 units in the 7^{th} year. Assuming that the production increases uniformly by a fixed number every year, find:

(i) the production in the first year.

(ii)the production in the 10^{th} year.

(iii)the total production in 7 years.

Ques15- If the 8^{th} term of an A.P. is 37 and the 15^{th} term is 15 more than the 12^{th} term, find the A.P.

THE END