GIRLS' HIGH SCHOOL AND COLLEGE, PRAYAGRAJ
Session 2020-21
CLASS–X (A,B,C,D,E,F)
SUBJECT–MATHEMATICS
WORKSHEET NO.-6

INSTRUCTIONS: – Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and websites given below.

NOTE – 1. Concise Mathematics ICSE Class X by R.K. Bansal

2. Understanding ICSE Mathematics Class X by M.L. Aggarwal

3. www.extramarks.com , www.topperlearning.com

Topic – Matrices

INTRODUCTION:

A Matrix is a rectangular arrangement of numbers, arranged in rows and column.

e.g. $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$, $\begin{bmatrix} 5 & 3 & 2 \end{bmatrix}$, etc.

NOTE: 1. Plural of matrix is matrices.

2. Each number or entity in a matrix is called its elements.

3. In a matrix, the horizontal lines are called rows; whereas the vertical lines are called columns.

ORDER OF A MATRIX:

The order of a matrix = Number of rows in it x Number of columns in it; i.e. if a matrix has **m** number of rows and **n** number of columns, its order is written as

m x n and is read as **m** by **n**.

Consider the matrix $\begin{array}{c} \begin{bmatrix} 2 & 1 & 5 \\ 3 & -2 & 7 \end{bmatrix} \begin{array}{l} \leftarrow \text{1st row} \\ \leftarrow \text{2nd row} \end{array} \\ \begin{array}{ccc} \text{1st Column} & \text{2nd Column} & \text{3rd Column} \end{array} \end{array}$

It has 2 rows and 3 columns, hence its order = 2 × 3 (read as 2 by 3)

ELEMENTS OF A MATRIX:

Each number or entity in a matrix is called its element.

Consider matrix $A = \begin{bmatrix} 2 & 4 & -3 \\ 0 & 1 & 2 \end{bmatrix}$

Since, matrix A has 2 rows and 3 columns, so the number of elements in it

= 2 * 3 = 6

TYPES OF MATIRICES:

1. Row Matrix: A matrix has only one row is called a row matrix.

$$\text{e.g. } \begin{array}{cc} \begin{bmatrix} a & \quad & b \end{bmatrix} & \leftarrow \text{Single row} \end{array}$$

$$\begin{array}{cc} \text{1st Column} & \text{2nd Column} \end{array}$$

Since, this matrix has 1 row and 2 columns, its order = 1 × 2 (1 by 2)

A row matrix is also called a row vector.

2. Column Matrix: A matrix which has only one column is called a column matrix.

$$\text{e.g. } \begin{bmatrix} a \\ b \end{bmatrix} \begin{array}{l} \leftarrow \text{1st row} \\ \leftarrow \text{2 nd row} \end{array}$$

$\uparrow$

Single Column

Since, this matrix has 2 rows and 1 column, its order = 2 × 1 (2 by 1)

A column matrix is also called a column vector.

3. Square Matrix: A matrix which has an equal number of rows and columns is called a square matrix.

$$\text{e.g. } \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{array}{l} \leftarrow \text{1 st row} \\ \leftarrow \text{2 nd row} \end{array}$$

$$\begin{array}{cc} \uparrow & \uparrow \\ \text{1 st Column} & \text{2 nd Column} \end{array}$$

Since, this matrix has 2 rows and 2 columns, its order = (2×2) (2 by 2)

4. Rectangular Matrix: A matrix in which the number of rows are not equal to the number of columns is called a rectangular matrix.

e.g. $\begin{bmatrix} 2 & 4 & 7 \\ 1 & 0 & 5 \end{bmatrix}$

Order is 2×3

5. Zero or Null Matrix: If each element of a matrix is zero, it is called a zero matrix or null matrix.

e.g. $\begin{bmatrix} 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ etc.

6. Diagonal Matrix: A square matrix which has all its elements zero each except those on the leading (or, principal) diagonal is called a diagonal matrix.

e.g. $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, etc.

NOTE: In a square matrix, the leading (principal) diagonal means the diagonal from top left to bottom right.

7. Unit or Indentity Matrix: A diagonal matrix in which each element of its leading diagonal is unity (i.e. 1) is called a unit or identity matrix. It is denoted by I. In other words, it is a square matrix in which each element of its leading diagonal is equal to 1 and all other remaining elements of the matrix are zero each.

e.g. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, etc.

TRANSPOSE OF A MATRIX:

Transpose of a matrix is the matrix obtained on interchanging its rows and columns. If A is a matrix, then its transpose is denoted by A^t.

e.g. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 7 \end{bmatrix}$, then its transpose

$$A^t = \begin{bmatrix} 2 & 0 \\ 3 & 4 \\ 1 & 7 \end{bmatrix}$$

EQUALITY OF MATRICES:

Two matrices are said to be equal if:

(i) both the matrices have the same order,

(ii) the corresponding elements of both the matrices are equal.

i.e. if $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$; then $A = B$

ADDITION OF MATRICES:

Compatibility for addition of matrices:

Two matrices can be added together, if they are of the same order.

To add two matrices of the same order means to add corresponding elements of both the matrices.

e.g. If $A = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$$A + B = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2+3 & 1+2 \\ 5+1 & 6+4 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 6 & 10 \end{bmatrix}$$

SUBTRACTION OF MATRICES:

The same rule and method is used for subtraction of matrices as is used for the addition of matrices.

The $A = \begin{bmatrix} 5 & 4 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 4 & 2 \end{bmatrix}$

then $A - B = \begin{bmatrix} 5 & 4 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 5-3 & 4-0 \\ 2-4 & 1-2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -2 & -1 \end{bmatrix}$

Multiplication of a matrix by a scalar (real number):

To multiply a matrix by a scalar means to multiply each of its elements by this scalar.

e.g. $3[4 \quad 1] = [3 * 4 \quad 3 * 1] = [12 \quad 3]$

Multiplication of Matrices:

Compatibility for Multiplication of Matrices:

Two matrices A and B can be multiplied together to get the product matrix AB if, and only if, the number of columns in A (the left hand matrix) is equal to the number of rows in B (the right hand matrix).

Let matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Since, the number of columns in $A =$ the number of rows in $B = 2$.

$\therefore$ Product matrix AB is possible.

And, $AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Step1: Multiply every element of 1st row of matrix A with corresponding element of 1st column of B and add them to get the first element of the 1st row of the product matrix AB.

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 * 1 + 4 * 3 & \\ & \end{bmatrix}$$

$$= \begin{bmatrix} 15 & \end{bmatrix}$$

Step2: Multiply every element of 1st row of matrix A with corresponding elements of 2nd column of B and add them to get the second element of the 1st row of product matrix AB.

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 3*2+4*4 \\ & \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 22 \\ & \end{bmatrix}$$

Step3: In the similar manner, multiply the elements of 2nd row of A with corresponding elements of the 1st column of B and get the first element of the second row of AB.

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 22 \\ 5*1+0*3 & \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 22 \\ 5 & \end{bmatrix}$$

Step4: Finally, multiply the elements of the second row of matrix A with corresponding elements of second column of matrix B to get the second element of the second row of AB.

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 22 \\ 5 & 2*5+0*4 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 22 \\ 5 & 10+0 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 22 \\ 5 & 10 \end{bmatrix}$$

$\therefore$ Product of matrices A and $B = AB$

$$= \begin{bmatrix} 1st\ row\ of\ A * 1st\ column\ of\ B & 1st\ row\ of\ A * 2nd\ column\ of\ B \\ 2nd\ row\ of\ A * 1st\ column\ of\ B & 2nd\ row\ of\ A * 2nd\ column\ of\ B \end{bmatrix}$$

Example1: If $A = \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix}$; $B = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$; find:

(i) $A + C$ (ii) $B - A$ (iii) $A + B - C$

Solution:

(i) $$A + C = \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5-3 & 4+2 \\ 3+1 & -1+0 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & -1 \end{bmatrix}$$

(ii) $$B - A = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-5 & 1-4 \\ 0-3 & 4+1 \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ -3 & 5 \end{bmatrix}$$

(iii) $$A + B - C = \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 5 \\ 3 & 3 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$$ [Evaluating $A + B$ then subtraction C]

$$= \begin{bmatrix} 7+3 & 5-2 \\ 3-1 & 3-0 \end{bmatrix} = \begin{bmatrix} 10 & 3 \\ 2 & 3 \end{bmatrix}$$

Example2: Given $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$,

find $A + 2B - 3C$.

Solution:

$$A + 2B - 3C = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + 2\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix}$$

Example3: If $A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$, evaluate $A^2 - 3A + 2I$, where I is a unit matrix of order 2.

Solution:

A unit matrix of order 2 means; a unit matrix of order 2 × 2.

Here, $A^2 = A \cdot A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 4+1 & -2-3 \\ -2-3 & 1+9 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix}$

therefore $A^2 - 3A + 2I = \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix} - 3\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix} - \begin{bmatrix} 6 & -3 \\ -3 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5-6+2 & -5+3+0 \\ -5+3+0 & 10-9+2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$$

Example4: Let $A = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$; find:

(i) $(A+B)(A-B)$ (ii) $A^2 - B^2$ Is $(A+B)(A-B) = A^2 - B^2$?

Solution:

(i) $\because \quad A + B = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3+1 & 2+0 \\ 0+1 & 5+2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 7 \end{bmatrix}$

and, $A - B = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3-1 & 2-0 \\ 0-1 & 5-2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$

$$\therefore (A+B)(A-B) = \begin{bmatrix} 4 & 2 \\ 1 & 7 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4*2+2*-1 & 4*2+2*3 \\ 1*2+7*-1 & 1*2+7*3 \end{bmatrix} = \begin{bmatrix} 6 & 14 \\ -5 & 23 \end{bmatrix}$$

(ii) $\because \quad A^2 = A * A = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 3*3+2*0 & 3*2+2*5 \\ 0*3+5*0 & 0*2+5*5 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & 25 \end{bmatrix}$$

and, $B^2 = B * B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 1*1+0*1 & 1*0+0*2 \\ 1*1+2*1 & 1*0+2*2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$$

$$\therefore \quad A^2 - B^2 = \begin{bmatrix} 9 & 16 \\ 0 & 25 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 16 \\ -3 & 21 \end{bmatrix}$$

From the results of parts (i) and (ii), it is clear that:

$$(A+B)(A-B) \neq A^2 - B^2$$

SOLVE THE FOLLOWING QUESTIONS:

Question 1 – Given $A = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix}$, and $B = \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix}$; find the matrix X in each of the following:

(i) $A + X = B$

(ii) $A - X = B$

(iii) $X - B = A$

Question 2 – Find, x and y from the following equations:

(i) $\begin{bmatrix} 5 & 2 \\ -1 & y-1 \end{bmatrix} - \begin{bmatrix} 1 & x-1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$

(ii) $[-8 \quad x] + [y \quad -2] = [-3 \quad 2]$

Question 3 – Given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$

(i) find the matrix $2A + B$.

(ii) find a matrix C such that:

$$C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Question 4 – Given $A = \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$ and A^t is its transpose matrix. Find:

(i) $2A + 3A^t$ (ii) $2A^t - 3A$

(iii) $\frac{1}{2}A - \frac{1}{3}A^t$ (iv) $A^t - \frac{1}{3}A$

Question 5 – If I is the unit matrix of order 2 × 2; find the matrix M, such that:

(i) $M - 2I = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix}$ (ii) $5M + 3I = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix}$

Question 6 – If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, find the matrix M.

Question 7 – Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$.

Find $A^2 + AC - 5B$.

Question 8 – Solve for x and y:

$$\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}.$$

Question 9 – If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$ find: $AB - 5C$.

Question 10 – Given $A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, find the matrix X such that; $A + X = 2B + C$.

Question 11 – If matrix $X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ and $2X - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$, find the matrix $'X'$ and matrix $'Y'$.

Question 12 – If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ and I is the identity matrix of the same order and A^t is the transpose of matrix A, find $A^t \cdot B + BI$.

Question 13 – Let $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$.

Find $A^2 - A + BC$

Question 14 – Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$. Find $A^2 + AB + B^2$.

Question 15 – Given $A = \begin{bmatrix} P & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ and $BA = C^2$.

Find the values of P and q.

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