GIRLS' HIGH SCHOOL AND COLLEGE, PRAYAGRAJ
Session 2020-21
CLASS–X (A,B,C,D,E,F)
SUBJECT–MATHEMATICS
WORKSHEET NO.-5

INSTRUCTIONS: – Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and websites given below.

NOTE – 1. Concise Mathematics ICSE Class X by R.K. Bansal

2. Understanding ICSE Mathematics Class X by M.L. Aggarwal

3. www.extramarks.com , www.topperlearning.com

Topic – Remainder and factor theorem

Polynomials in one variable

An expression of the form $a_0 + a_1x + a_2x^2 + \cdots \ldots \ldots \ldots . + a_{n-1}x^{n-1} + a_nx^n$, where $a_{0,} a_1, a_2, \ldots \ldots \ldots . . a_{n-1}, a_n$ are real numbers, $a_n \neq 0$ and n is a non negative integer, is called a polynomial in x of degree n.

CONSTANTS – A symbol having a fixed numerical value is called a constant.

Examples – $9,\ -6,\ \frac{4}{7},\ \sqrt{2},\ \pi$ are all constants.

VARIABLES – A symbol which may be assigned different numerical values is known as a variable.

Example – We know that the circumference of a circle is given by the formula $C = 2\pi r$, where r is the radius of the circle.

Here, 2 and π are constants, while C and r are variables.

REMAINDER THEOREM:

If $f(x)$, a polynomial in x, is divided by $(x - a)$, the remainder $= f(a)$.

e.g. If $f(x)$ is divided by $(x - 3)$, the remainder is $f(3)$.

For Finding the Remainder, using Remainder Theorem:

Step 1: Put the divisor equal to zero and solve the equation obtained to get the value of its variable.

Step 2: Substitute the value of the variable, obtained in step 1, in the given polynomial and simplify it to get the required remainder.

FACTOR THEOREM:

When a polynomial $f(x)$ is divided by $x-a$, the remainder $= f(a)$. And if remainder $f(a) = 0$; $x - a$ is a factor of the polynomial $f(x)$.

USING THE FACTOR THEOREM TO FACTORISE THE GIVEN POLYNOMIAL:

Factorising a polynomial completely after obtaining one factor by factor theorem.

When expression $f(x)$ is divided by $x-a$, the remainder $= f(a)$.

If the remainder $f(a) = 0$

$\Rightarrow x - a$ is a factor of expression $f(x)$.

Conversely, if for the expression $f(x)$, $f(a) = 0$; $\Rightarrow$ $(x-a)$ is a factor.

For example:

Let $f(x) = x^2 - 7x + 10$; then

$f(2) = (2)^2 - 7 * 2 + 10 = 0$

$\Rightarrow x - 2$ is a factor of $f(x) = x^2 - 7x + 10$

Example1: Find the value of $'a'$ if the division of $ax^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder of 5.

Solution:

$$x + 3 = 0 \quad \Rightarrow \quad x = -3$$

Given, remainder is 5; therefore:

The value of $ax^3 + 9x^2 + 4x - 10$ at $x = -3$ is 5

$$\Rightarrow a(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$$
$$\Rightarrow -27a + 81 - 12 - 10 = 5$$
$$\Rightarrow -27a + 81 - 22 = 5$$
$$\Rightarrow -27a = 5 + 22 - 81$$
$$\Rightarrow -27a = 27 - 81$$
$$\Rightarrow -27a = -54$$
$$\Rightarrow a = 2$$

Example2: If $x - 2$ is a factor $x^2 - 7x + 2a$, find the value a.

Solution:

$$x - 2 = 0 \quad \Rightarrow \quad x = 2$$

Since, $x - 2$ is a factor of polynomial $x^2 - 7x + 2a$

$\Rightarrow$ Remainder = 0 $\Rightarrow$ $(2)^2 - 7(2) + 2a = 0$

$$\Rightarrow \quad 4 - 14 + 2a = 0$$
$$\Rightarrow -10 = -2a$$
$$\Rightarrow a = 5$$

Example3: Show that $2x + 7$ is a factor of $2x^3 + 5x^2 - 11x - 14$. Hence, factorise the given expression completely, using the factor theorem.

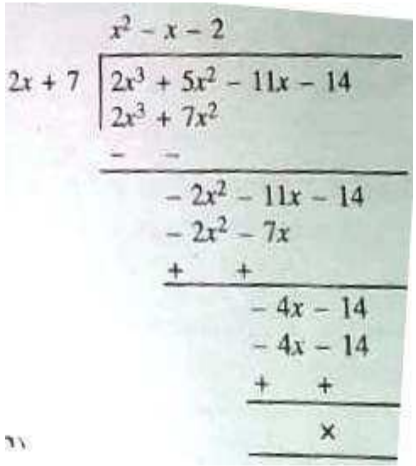
Solution:

$$2x + 7 = 0 \quad \Rightarrow \quad x = -\frac{7}{2}$$

Remainder = Value of $2x^3 + 5x^2 - 11x - 14$ at $x = -\frac{7}{2}$

$$= 2\left(-\frac{7}{2}\right)^3 + 5\left(-\frac{7}{2}\right)^2 - 11\left(-\frac{7}{2}\right) - 14$$
$$= -\frac{343}{4} + \frac{245}{4} + \frac{77}{2} - 14$$

$= \frac{-343+245+154-56}{4} = 0$



$\Rightarrow (2x+7)$ is a factor of $2x^3+5x^2-11x-14$

$\therefore\ 2x^3+5x^2-11x-14 = (2x+7)(x^2-x-2)$

$= (2x+7)(x^2-2x+x-2)$

$= (2x+7)[x(x-2)+1(x-2)]$

$= (2x+7)(x-2)(x+1)$

Example4: Find the values of $'a'$ and $'b'$ so that the polynomial

$x^3+ax^2+bx-45$ has $(x-1)$ and $(x+5)$ as its factors.

For the values of $'a'$ and $'b'$, as obtained above, factorise the given polynomial completely.

Solution:

$(x-1)$ is a factor of given polynomial $x^3+ax^2+bx-45$

$\Rightarrow \quad (1)^3+a(1)^2+b(1)-45=0 \qquad [x-1=0 \Rightarrow x=1]$

i.e. $\quad a+b=44 \qquad$ ……I

$(x+5)$ is a factor of given polynomial

$\Rightarrow \quad (-5)^3+a(-5)^2+b(-5)-45=0 \qquad [x+5=0 \ \Rightarrow \ x=-5]$

$\Rightarrow \quad -125+25a-5b-45=0$

i.e. $\quad 5a-b=34 \qquad$ …….II

On solving equations I and II. We get :

$a=13$ and $b=31$

$\therefore$ The given polynomial $x^3+ax^2+bx-45$

$= x^3+13x^2+31x-45$

Now divide this polynomial

by $(x-1)$ as shown alongside:

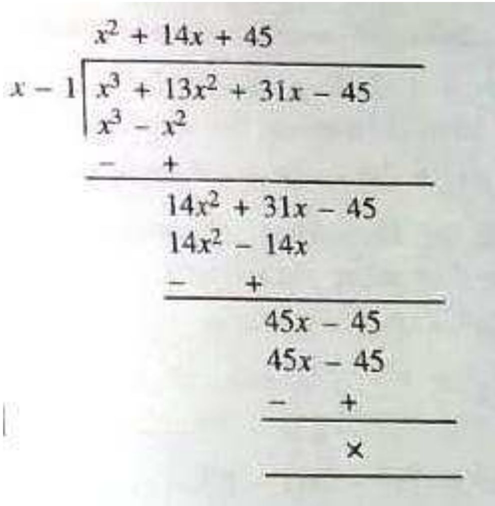
$\therefore\ x^3+13x^2+31x-45$

$=(x-1)(x^2+14x+45)$

$=(x-1)(x^2+9x+5x+45)$

$=(x-1)\ [x(x+9)+5\ (x+9)]$

$=(x-1)\ (x+9)\ (x+5)$



SOLVE THE FOLLOWING QUESTIONS:

Question 1 – If $2x+1$ is a factor of $2x^2+ax-3$, find the value of a.

Question 2 – Find the values of constants a and b when $x-2$ and $x+3$ both are the factors of expression $x^3+ax^2+bx-12$.

Question 3 – Find the value of k, if $2x+1$ is a factor of $(3k+2)\ x^3+(k-1)$.

Question 4 – Find the value of a, if $x-2$ is a factor of

$$2x^5-6x^4-2ax^3+6ax^2+4ax+8.$$

Question 5 – Find the values of m and n so that $x-1$ and $x+2$ both are factors of

$$x^3+(3m+1)x^2+nx-18.$$

Question 6 – If x^3+ax^2+bx+6 has $x-2$ as a factor and leaves a remainder 3 when divided by $x-3$, find the values of a and b.

Question 7 – What number should be subtracted from $x^3 + 3x^2 - 8x + 14$ so that on dividing it by $x - 2$, the remainder is 10?

Question 8 – The polynomials $2x^3 - 7x^2 + ax - 6$ and

$x^3 - 8x^2 + (2a + 1)x - 16$ leave the same remainder when divided by $x - 2$. Find the value of $'a'$.

Question 9 – If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b.

Question 10 – Find $'a'$ if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leave the same remainder when divided by $x + 3$.

Question 11 – Using the Factor Theorem, show that:

$(x + 5)$ is a factor of $2x^3 + 5x^2 - 28x - 15$. Hence, factorise the expression $2x^3 + 5x^2 - 28x - 15$ completely.

Question 12– Using the Remainder Theorem, factorise the expression:

$$2x^3 + x^2 - 13x + 6$$

Question 13–Using the Remainder Theorem, factorise the expression $3x^3 + 10x^2 + x - 6$. Hence, solve the equation $3x^3 + 10x^2 + x - 6 = 0$.

Question 14 – Given that $x-2$ and $x+1$ are factors of $f(x)=x^3+3x^2+ax+b$; calculate the values of a and b. Hence, find all the factors of $f(x)$.

Question 15 – If $x+a$ is a common factor of expressions $f(x)=x^2+px+q$ and $g(x)=x^2+mx+n$; show that : $a=\frac{n-q}{m-p}$

Question 16 – The polynomials ax^3+3x^2-3 and $2x^3-5x+a$, when divided by $x-4$, leave the same remainder in each case. Find the value of a.

Question 17 – Find the value of $'a'$, if $(x-a)$ is a factor of x^3-ax^2+x+2.

Question 18 – Find the number that must be subtracted from the polynomial $3y^3+y^2-22y+15$, so that the resulting polynomial is completely divisible by $y+3$.

Question 19 – Factorise the expression

$$f(x)=2x^3-7x^2-3x+18.$$

Hence, find all possible values of x for which $f(x)=0$

Question 20 – The expression $4x^3-bx^2+x-c$ leaves remainders 0 and 30 when divided by $x+1$ and $2x-3$ respectively. Calculate the values of b and c. Hence, factorise the expression completely.

Question 21 – Find, in each case, the remainder when:

(i) x^4-3x^2+2x+1 is divided by $x-1$.

(ii) $x^3+3x^2-12x+4$ is divided by $x-2$.

(iii) $x^4 + 1$ is divided by $x + 1$.

Question 22 – Show that:

(i) $x - 2$ is a factor of $5x^2 + 15x - 50$.

(ii) $3x + 2$ is a factor of $3x^2 - x - 2$.

Question 23 – When $x^3 + 2x^2 - kx + 4$ is divided by $x - 2$, the remainder is k. Find the value of constant k.

Question 24 – Find the value of a, if the division of $ax^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder 5.

Question 25 – The expression $2x^3 + ax^2 + bx - 2$ leaves remainder 7 and 0 when divided by $2x - 3$ and $x + 2$ respectively. Calculate the values of a and b.

-------THE END-------