GIRLS' HIGH SCHOOL AND COLLEGE, PRAYAGRAJ
Session 2020-21
CLASS–X (A,B,C,D,E,F)
SUBJECT–MATHEMATICS
WORKSHEET NO.-4

INSTRUCTIONS: – Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and websites given below.

NOTE – 1. Concise Mathematics ICSE Class X by R.K. Bansal

2. Understanding ICSE Mathematics Class X by M.L. Aggarwal

3. www.extramarks.com , www.topperlearning.com

Topic – Quadratic Equations

INTRODUCTION– An equation with one variable, in which the highest power of the variable is two, is known as quadratic equation. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are all real numbers and $a \neq 0$

e.g. $4x^2 + 5x - 6 = 0$ is a quadratic equation in standard form.

NOTE: 1. Every quadratic equation gives two values of the unknown variable used in it and these values are called roots of the equation.

2. Discriminant : For the quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$; the expression $b^2 - 4ac$ is called discriminant and is, in general, denoted by the letter 'D'.

Thus, discriminant $D = b^2 - 4ac$

3. If a quadratic equation contains only two terms one square term and one first power term of the unknown, it is called adfected quadratic equation.

For eg: $4x^2 + 5x = 0$

4. If the quadratic equation contains only the square of the unknown, it is called pure quadratic equation.

For eg: $x^2 = 4$

TO EXAMINE THE NATURE OF THE ROOTS:

Examining the roots of a quadratic equation means to know the type of its roots i.e. whether they are real or imaginary, rational or irrational, equal or unequal.

The nature of the roots of a quadratic equation depends entirely on the values of its discriminant $b^2 - 4ac$.

If for a quadratic equation $ax^2 + bx + c = 0$; where a, b and c are real numbers and $a \neq 0$, then discriminant:

(i) $b^2 - 4ac = 0 \Rightarrow$ the roots are real and equal.

(ii) $b^2 - 4ac > 0 \Rightarrow$ the roots are real and unequal.

(iv) $b^2 - 4ac < 0 \Rightarrow$ the roots are imaginary (not real).

SOLVING QUADRATIC EQUATIONS BY FACTORISATION:

Steps:

(i) Clear all fractions and brackets, if necessary.

(ii) Transpose all the terms to the left hand side to get an equation in the form $ax^2 + bx + c = 0$.

(iii) Factorise the expression on the left hand side.

(iv) Put each factor equal to zero and solve.

SOLVING QUADRATIC EQUATIONS USING THE FORMULA:

The roots of the quadratic equation $ax^2 + bx + c = 0$; where $a \neq 0$ can be obtained by using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

FOR SOLVING A WORD PROBLEM BASED ON QUADRATIC EQUATION ADOPT THE FOLLOWING STEPS:

1. Represent the unknown quantity of the problem by variable $'x'$.
2. Translate the given statement to form an equation in terms of $'x'$.
3. Solve the equation.

Example1: Find the value of $'m'$, if the roots of the following quadratic equation are equal:

$$(4+m)x^2+(m+1)x+1=0$$

Solution:

For the given equation $(4+m)x^2+(m+1)x+1=0$

$a=4+m$, $b=m+1$ and $c=1$

Since, the roots are equal

$\therefore\ b^2-4ac=0 \Rightarrow (m+1)^2-4(4+m)*1=0$

$$\Rightarrow m^2+2m+1-16-4m=0$$

$$\Rightarrow m^2-2m-15=0$$

$$\Rightarrow m^2-5m+3m-15=0$$

$$\Rightarrow m(m-5)+3(m-5)=0$$

$$\Rightarrow (m-5)\,(m+3)=0$$

$$\Rightarrow m=5 \quad or \quad m=-3$$

Example2: $\frac{x}{x-1}+\frac{x-1}{x}=2\frac{1}{2}$

Solution:

$$\frac{x}{x-1}+\frac{x-1}{x}=2\frac{1}{2}$$

$$\Rightarrow \quad \frac{x}{x-1} + \frac{x-1}{x} = \frac{5}{2}$$

$$\Rightarrow \quad \frac{x^2+(x-1)^2}{x(x-1)} = \frac{5}{2}$$

$$\Rightarrow \quad \frac{x^2+x^2+1-2x}{x(x-1)} = \frac{5}{2}$$

$$\Rightarrow \quad \frac{2x^2-2x+1}{x(x-1)} = \frac{5}{2}$$

$$\Rightarrow \quad 2(2x^2 - 2x + 1) = 5x(x - 1)$$

$$\Rightarrow \quad 4x^2 - 4x + 2 = 5x^2 - 5x$$

$$\Rightarrow \quad 5x^2 - 4x^2 - 5x + 4x - 2 = 0$$

$$\Rightarrow \quad x^2 - x - 2 = 0$$

$$\Rightarrow \quad x^2 - 2x + x - 2 = 0$$

$$\Rightarrow \quad x(x - 2) + 1(x - 2) = 0$$

$$\Rightarrow \quad (x - 2)\ (x + 1) = 0$$

$$\Rightarrow \quad x = 2, \quad or \quad x = -1$$

Example3: Find the value of $'k'$ for which $x = 3$ is a solution of the quadratic equation,

$(k + 2)x^2 - kx + 6 = 0$ Hence, find the other root of the equation.

Solution:

$x = 3$ is a solution of equation $(k + 2)x^2 - kx + 6 = 0$

$$\Rightarrow \quad (k + 2) * 9 - k * 3 + 6 = 0$$

$$\Rightarrow \quad 9k + 18 - 3k + 6 = 0$$

$$\Rightarrow \quad 6k + 24 = 0$$

$$\Rightarrow \quad 6k = -24$$

$$\Rightarrow \quad k = -4$$

For $k = -4, \quad (k+2)x^2 - kx + 6 = 0$

$$\Rightarrow \quad (-4+2)x^2 - (-4)x + 6 = 0$$

$$\Rightarrow \quad -2x^2 + 4x + 6 = 0$$

$$\Rightarrow \quad x^2 - 2x - 3 = 0$$

$$\Rightarrow \quad x^2 - 3x + x - 3 = 0$$

$$\Rightarrow \quad x(x-3) + 1(x-3) = 0$$

$$\Rightarrow \quad (x-3)\ (x+1) = 0$$

$$\Rightarrow \quad x = 3 \quad or \ \ x = -1$$

Since, $x = 3$ is already given to be one root (solution) of the equation.

$\therefore$ The other root of the equation is $x = -1$

Example4: Solve each of the following equation for x and give, your answer correct to 2 decimal places:

$$x^2 - 10x + 6 = 0$$

$$\Rightarrow \quad a = 1\ ; b = -10 \ \ and \ \ c = 6$$

$$\therefore \quad b^2 - 4ac = (-10)^2 - 4(1)(6)$$

$$= 100 - 24$$

$$= 76$$

$$\sqrt{b^2 - 4ac} = \sqrt{76} = 8.716$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{10 \pm 8.718}{2*1}$$

$$= \frac{10+8.718}{2}\ , or \quad \frac{10-8.718}{2}$$

$= \; 9.359 \,, or \;\; 0.641$

$= \; 9.36 \,, or \;\; 0.64$ [Correct to 2 decimal places]

Example5: Solve the following equation:

$x - \frac{18}{x} = 6$, Give your answer to two significant figures.

Solution:

$$x - \frac{18}{6} = 6 \;\; \Rightarrow \;\; \frac{x^2 - 18}{x} = 6$$

$$\Rightarrow \;\; x^2 - 18 = 6x$$

$$\Rightarrow \;\; x^2 - 6x - 18 = 0$$

Comparing with $ax^2 + bx + c = 0$, we get : $a = 1, b = -6$ and $c = -18$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \;\; x = \frac{6 \pm \sqrt{36 - 4*(1)*(-18)}}{2*1}$$

$$\Rightarrow \;\; x = \frac{6 \pm \sqrt{36 + 72}}{2}$$

$$\Rightarrow \;\; x = \frac{6 \pm \sqrt{108}}{2}$$

$$\Rightarrow \;\; x = \frac{6 \pm 10.392}{2}$$

$$\Rightarrow \;\; x = \frac{6 + 10.392}{2} \;\; or \;\; \frac{6 - 10.392}{2}$$

$$\Rightarrow \;\; x = \frac{16.392}{2} \;\; or \;\; \frac{-4.392}{2}$$

$$\Rightarrow \;\; x = 8.196 \;\; or \;\; -2.196$$

$$\Rightarrow \;\; x = 8.2 \;\; or \;\; -2.2$$

Example6: Solve: $2x^4 - 5x^2 + 3 = 0$

Taking $x^2 = y$

$$\Rightarrow \;\; 2y^2 - 5y + 3 = 0$$

$\Rightarrow \quad 2y^2 - 2y - 3y + 3 = 0$

$\Rightarrow \quad 2y(y-1) - 3(y-1) = 0$

$\Rightarrow \quad (y-1)\ (2y-3) = 0$

$\Rightarrow \quad y = 1 \quad or \quad y = \frac{3}{2}$

When $y = 1 \Rightarrow x^2 = 1 \ \Rightarrow \ \ x = \pm 1$

and when $y = \frac{3}{2} \ \Rightarrow x^2 = \frac{3}{2} \ \Rightarrow x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{6}}{2}$

$\therefore$ Required solution $= \ 1, -1, \ \frac{\sqrt{6}}{2}\ , \ or \ \ -\frac{\sqrt{6}}{2}$

Example7: For the same amount of work, A takes 6 hours less than B. If together they complete the work in 13 hours 20 minutes; find how much time will B alone take to complete the work?

Solution:

If B alone takes x hours then A alone takes $(x-6)$ hours for the same work.

$\therefore \frac{1}{x-6} + \frac{1}{x} = \frac{3}{40} \quad [\because 13\ hrs.\ 20\ min. = \left(13 + \frac{20}{60}\right) hrs. = \frac{40}{3} hrs.]$

$\Rightarrow \qquad \frac{x+x-6}{(x-6)x} = \frac{3}{40}$

$\Rightarrow \qquad 3x^2 - 18x = 80x - 240$ i.e. $3x^2 - 98x + 240 = 0$

$\Rightarrow \qquad 3x^2 - 90x - 8x + 240 = 0$ i.e. $(x-30)(3x-8) = 0$

$\Rightarrow \qquad x = 30, \ \ or \ \ x = \frac{8}{3}$ i.e. $x = 30$

$\therefore$ B alone will take 30 hrs. to complete the work.

Example8: The length of a verandah is 3m more than its breadth. The numerical value of its area is equal to the numerical value of its perimeter.

(i) Taking $'x'$ as the breadth of the verandah, write an equation in $'x'$ that represents the above statement.

(ii) Solve the equation obtained in (i) above and hence find the dimensions of the verandah.

Solution:

Since breadth $= x\ m\ \therefore Length = (x+3)m$

(i) Given : Area of verandah = its perimeter [Numerically]

i.e. length * breadth = 2(length + breadth)

$\Rightarrow\ (x+3)\,.\,x = 2(x+3+x)$

$\Rightarrow\ x^2 + 3x = 4x + 6$

$\Rightarrow\ x^2 - x - 6 = 0$

(ii) $x^2 - x - 6 = 0$

$\Rightarrow\ (x-3)\,(x+2) = 0$ [On factorising]

$\Rightarrow\ x = 3,\ or\ x = -2$

Since, the breadth cannot be negative, $\therefore x = 3$

Hence, the length of verandah $= (x+3)m = (3+3)m = 6m$

and, its breadth $= x\ m = 3m$

Example9: By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car.

Solution:

Let the original speed of the car $= x$ km /hr

$\therefore$ Time taken by it to cover 72 km $= \frac{72}{x}$ hrs $\left[\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

New speed of the car $= (x+10)$ km/hr

$\therefore$ New time taken by the car to cover 72 km $= \frac{72}{x+10}$ hrs

Given : Time is reduced by 36 minutes :

$$\Rightarrow \frac{72}{x} - \frac{72}{x+10} = \frac{36}{60}$$

On solving, we get : $x = -40 \ \ or \ \ x = 30$

Since, speed cannot be negative hence the value of $x = 30$

The original speed of the car = 30 km/hr.

Example10: Five years ago, a woman's age was the square of her son's age. Ten years hence her age will be twice that or her son's age. Find:

(i) the age of the son five years ago.

(ii) the present age of the woman.

Solution:

Let the age of the son 5 years ago $= x$ years.

$\therefore$ The woman's age 5 years ago $= x^2$ years

$\Rightarrow$ The present age of woman's = $(x^2 + 5)$ years

and, the present age of her son $= (x + 5)$ years

10 years hence:

The woman's age will be $= (x^2 + 5) + 10$ years $= (x^2 + 15)$ years

and, her son's age will be $= (x + 5) + 10$ years $= (x + 15)$ years

According to the given statement:

10 years hence, woman's age = twice her son's age

$$\Rightarrow \quad x^2 + 15 = 2(x + 15)$$

$$\Rightarrow \quad x^2 - 2x - 15 = 0$$

On solving, we get : $x = 5 \ or \ x = -3$

Since, age cannot be negative, reject $x = -3$.

$\therefore \qquad x = 5$

(i) The age of the son 5 years ago $=$ x years = 5 years

(ii) The present age of the woman $=$ $(x^2 + 5)$ years $=$ $(5^2 + 5)$ years

= 30 years

Example11: A piece of cloth costs Rs 200. If the piece was 5 m longer and each metre of cloth costs Rs 2 less; the cost of the piece would have remained unchanged. How long is the piece and what is the original rate per metre?

Solution:

Let the length of the piece be x metre

Since, the cost of x metre cloth = Rs 200

$\Rightarrow$ Cost of each metre of cloth = Rs $\frac{200}{x}$

New length of cloth = $(x + 5)$m

New cost of each metre of cloth = Rs $\frac{200}{x+5}$

Given: $\frac{200}{x} - \frac{200}{x+5} = 2 \qquad i.e. \ \frac{200x+1000-200x}{x(x+5)} = 2$

$i.e. \qquad 2(x^2 + 5x) = 1000$

$\Rightarrow \qquad x^2 + 5x = 500$

$i.e. \qquad x^2 + 5x - 500 = 0$

$\Rightarrow \qquad x = -25, \ or \ x = 20$ [On Solving]

$\therefore x = 20 \quad \Rightarrow$ The length of the piece = 20m

And, the original rate per metre = Rs $\frac{200}{20} = Rs\ 10$

SOLVE THE FOLLOWING QUESTIONS:

Question 1 – Find the value of $'p'$, if the following quadratic equations have equal roots:

(i) $4x^2 - (p-2)x + 1 = 0$

Question 2 – Solve equation using factorization method:

$$\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$$

Question 3 – Solve equation using factorization :

$$\left(1 + \frac{1}{x+1}\right)\left(1 - \frac{1}{x-1}\right) = \frac{7}{8}$$

Question 4 – Solve the equation $2x - \frac{1}{x} = 7$. Write your answer correct to two decimal places.

Question 5 – Solve the following equation and give your answer correct to 3 significant figures:

$$5x^2 - 3x - 4 = 0$$

Question 6 – Solve: $2x^4 - 5x^2 + 3 = 0$

Question 7 – Use the substitution $y = 2x + 3$ to solve for x,

if $4(2x+3)^2 - (2x+3) - 14 = 0$.

Question 8 – A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car.

Question 9 – Some students planned a picnic. The budget for the food was Rs 480. As eight of them failed to join the party, the cost of the food for each member increased by Rs 10. Find, how many students went for the picnic?

Question 10 – A plane left 30 minutes later than the scheduled time and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed.

Question 11 – Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 5 km/hr faster than the second train. If after 2 hours, they are 50 km apart, find the speed of each train.

Question 12 – A shopkeeper buys a certain number of books for Rs 960. If the cost per book was Rs 8 less, the number of books that could be bought for Rs 960 would be 4 more. Taking the original cost of each book to be Rs x, write an equation in x and solve it.

Question 13 – The age of a father is twice the square of the age of his son. Eight years hence, the age of the father will be 4 years more than three times the age of the son. Find their present ages.

Question 14 – The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the original point in 4 hours 30 minutes. Find the speed of the stream.

Question 15 – Some school children went on an excursion by a bus to a picnic spot at a distance of 300 km. while returning, it was raining and the bus had to reduce its speed by 5 km/hr and it took two hours longer for returning. Find the time taken to return.

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