GIRLS' HIGH SCHOOL AND COLLEGE, PRAYAGRAJ

WORKSHEET-5

CLASS 9 (A,B,C,D,E&F)

SESSION 2020-2021

SUBJECT-MATHEMATICS

INSTRUCTIONS:- Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and website referred and thereafter answer the given questions.

Note:- 1. Student should refer to books of class 6, 7 & 8 for reference and also the following websites : www.extramarks.com and www.topperlearning.com

2. Concise MATHEMATICS I.C.S.E. Class-IX by R.K. Bansal

3. Understanding I.C.S.E. MATHEMATICS class-IX by M.L. Aggarwal

TOPIC–FACTORISATION

What is a Factor – When a polynomial (an algebraic expression) is expressed as the product of two or more expressions, each of the expressions is called a factor of the polynomial. e.g. $x^2 + 5x + 6 = (x + 2)(x + 3)$.
i.e. $(x + 2)(x + 3)$ are the factors of $x^2 + 5x + 6$

What is Factorisation – The process of writing an expression in the form of terms or brackets multiplied together is called factorisation. Each term and each bracket is called a factor of the expression. e.g. $5x^2 + 15 = 5\,(x^2 + 3)$ i.e. 5 and $(x^2 + 3)$ are the factors of $5x^2 + 15$.

METHODS OF FACTORISATION

TYPE 1 – TAKING OUT THE COMMON FACTORS (DIRECT METHOD)

When each term of a given expression contains a common factor, divide each term by this factor and enclose the quotient within brackets keeping the common factor outside the bracket. The terms of this expression are $6a^2$ and $-3ax$. The HCF of these two terms is 3a.

Therefore, $6a^2 - 3ax = 3a\,(\frac{6a^2}{3a} - \frac{3ax}{3a}) = 3a\,(2a - x)$

Solved Example

Factorise: $8ab^2 + 12a^2b$

It can easily be seen that 4ab is the largest expression, which divides both the terms $8ab^2$ and $12a^2b$ completely.
$8ab^2 + 12a^2b = 4ab(2b + 3a)$
Therefore the factors of the expression $8ab^2 + 12a^2b$ *are* $4ab$ *and* $(2b + 3a)$

Q 1. Factorise by taking out the common factor

(i) $2(2x - 5y)(3x + 4y) - 6(2x - 5y)(x - y)$
(ii) $xy(3x^2 - 2y^2) - yz(2y^2 - 3x^2) + zx(15x^2 - 10y^2)$
(iii) $ab(a^2 + b^2 - c^2) - bc(c^2 - a^2 - b^2) + ca(a^2 + b^2 - c^2)$
(iv) $2x(a - b) + 3y(5a - 5b) + 4z(2b - 2a)$
(v) $4(x + y)^2 - 3(x + y)$
(vi) $x(a - 5) + y(5 - a)$

TYPE 2 – GROUPING METHOD

An expression of an even number of terms can be resolved into factors, if the terms are arranged in groups such that each group has a common factor.

Procedure

(a) Group the terms of the given expression in such a way that each group has a common factor.

(b) Factorise each of the groups formed.

(c) From each group obtained in step 2, take out the common factor and put the other factor into a bracket.

Solved Example

1. Factorise: $ab + bc + ax + cx$

(a) Group the common terms together (ab + bc) + (ax + cx) forming group

(b) b (a + c) + x (a + c) taking out common factors from each group

(c) (a + c) (b + x) taking (a + c) as common factor.

(d) Therefore the factors of $ab + bc + ax + cx$ will be (a + c) (b + x)

2. Factorise: $a^2 + \frac{1}{a^2} + 2 - 5a - \frac{5}{a}$

$$a^2 + \frac{1}{a^2} + 2 - 5a - \frac{5}{a} = (a^2 + \frac{1}{a^2} + 2) - 5(a + \frac{1}{a})$$
$$= \left(a + \frac{1}{a}\right)^2 - 5\left(a + \frac{1}{a}\right) = \left(a + \frac{1}{a}\right)\left(a + \frac{1}{a}\right) - 5\left(a + \frac{1}{a}\right)$$
$$= \left(a + \frac{1}{a}\right)\left(a + \frac{1}{a} - 5\right)$$

Q 2. Factorise by grouping method

(i) $a^3 + a - 3a^2 - 3$
(ii) $a^4 - 2a^3 - 4a + 8$
(iii) $ab\,(x^2 + 1) + x\,(a^2 + b^2)$
(iv) $(ax + by)^2 + (bx - ay)^2$
(v) $(2a - b)^2 - 10a + 5b$
(vi) $a^2x^2 + (ax^2 + 1)\,x + a$
(vii) $y^2 - (a + b)y + ab$
(viii) $x^2 + y^2 + x + y + 2xy$
(ix) $m\,(x - 3y)^2 + n\,(3y - x) + 5x - 15\,y$
(x) $x\,(6x - 5y) - 4\,(6x - 5y)^2$

TYPE 3 – BY SPLITTING THE MIDDLE TERM (TRINOMIAL OF THE FORM $ax^2 \pm bx \pm c$)

When a trinomial is of the form $ax^2 \pm bx \pm c$ *or* $a + bx + cx^2$, split *b* (the coefficient of x in the middle term) into two parts such that the sum of these two parts is equal to *b* and the product of these two parts is equal to the product of a and c. Then factorise by grouping method.

Solved Examples

1. Factorise: $x^2 + 5x + 6$

   - $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$ $since, 3 + 2 = 5\ and\ 3 \text{ x } 2 = 6$
   - $x\,(x + 3) + 2\,(x + 3) = (x + 2)\,(x + 3)$

2. Factorise: $x^2 - 5x + 6$

   - $x^2 - 5x + 6 = x^2 - 3x - 2x + 6$ $since, -3 + (-2) = -5\ and (-3) \text{ x} (-2) = 6$
   - $x\,(x - 3) - 2\,(x - 3) = (x - 2)\,(x - 3)$

3. Factorise: $x^2 - 5x - 6$

   - $x^2 - 5x - 6 = x^2 - 6x + x - 6$ $since, -6 + 1 = -5\ and - 6 \text{ x } 1 = -6$
   - $x\,(x - 6) + 1(x - 6) = (x - 6)\,(x + 1)$

4. Factorise: $x^2 + 5x - 6$

- $x^2 + 5x - 6 = x^2 + 6x - x - 6$ $\quad$ $since, 6 + (-1) = 5$ and $6 x(-1) = -6$
- $x(x + 6) - 1(x + 6) = (x + 6)(x - 1)$

5. Factorise: $2x^2 - 7x + 6$

- $2x^2 - 7x + 6 = 2x^2 - 4x - 3x + 6$ $\quad$ $since, -3 + (-4) = -7$ $and (-3) x(-4) = 12$
- $2x(x - 2) - 3(x - 2) = (2x - 3)(x - 2)$

6. Factorise : $6 + 11x + 3x^2$

- $6 + 11x + 3x^2 = 6 + 9x + 2x + 3x^2$ $\quad$ $since, 9 + 2 = 11$ and $9 x 2 = 18$
- $3(2 + 3x) + x(2 + 3x) = (2 + 3x)(3 + x)$

Q 3. Factorise by splitting the middle term

(i) $a^2 + 10a + 24$
(ii) $6a^2 - a - 15$
(iii) $a(3a - 2) - 1$
(iv) $(2a + b)^2 - 6a - 3b - 4$
(v) $3a^2 - 1 - 2a$
(vi) $(3x - 2y)^2 + 3(3x - 2y) - 10$
(vii) $5 - (3a^2 - 2a)(6 - 3a^2 + 2a)$
(viii) $24a^3 + 37a^2 - 5a$
(ix) $(x^2 - 3x)(x^2 - 3x - 1) - 20$
(x) $\frac{1}{35} + \frac{12}{35}a + a^2$

Important Note: $ax^2 + bx + c$, where a, b and c are real numbers is known as a trinomial or a quadratic expression in which a= coefficient of x^2, b= coefficient of x and c=a constant.

If we find the value of $b^2 - 4ac$ and this value is a perfect square, the trinomial $ax^2 + bx + c$ Is factorisable, otherwise not.

Solved Examples

1. Is $5x^2 + 17x + 6$ factorisable. If yes, factorise it.

- Comparing $5x^2 + 17x + 6$ with $ax^2 + bx + c$, we get a=5, b=17 and c=6
- Therefore, $b^2 - 4ac = (17)^2 - (4x5x6) = 289 - 120 = 169$ which is a perfect square.
- Thus, $5x^2 + 17x + 6$ is factorisable.

Now, $5x^2 + 17x + 6 = 5x^2 + 15x + 2x + 6 = 5x(x + 3) + 2(x + 3) = (x + 3)(5x + 2)$

2. Is $3x^2 - 8x - 15$ factorisable. If yes, factorise it.

- Comparing $3x^2 - 8x - 15$ with $ax^2 + bx + c$, we get a=3, b= - 8 and c= - 15
- Therefore, $b^2 - 4ac = (-8)^2 - (4 \times 3 \times (-15)) = 64 + 180 = 244$,which is not a perfect square.
- Thus, $3x^2 - 8x - 15$ is not factorisable.

Q 4. For each trinomial (quadratic equation) given below, find whether it is factorisable or not. Factorise if possible.

(i) $x^2 - 3x - 54$
(ii) $2x^2 - 7x - 15$
(iii) $2x^2 + 2x - 75$
(iv) $3x^2 + 4x - 10$
(v) $x(2x - 1) - 1$

TYPE 4 – DIFFERENCE OF TWO SQUARES

Since, $(x + y)(x - y) = x^2 - y^2$, the factors of $x^2 - y^2$ *are* $(x + y)$ *and* $(x - y)$

Solved Examples

Factorise :

(i) $x^2 - 25$ (ii) $9(x - y)^2 - (x + 2y)^2$ (iii) $48x^3 - 27x$

(iv) $16a^4 - b^4$ (v) $(1 - x^2)(1 - y^2) + 4xy$ (vi) $(p^2 + q^2 - r^2)^2 - 4p^2q^2$

Solution:

(i) $x^2 - 25 = (x - 5)(x + 5)$

(ii) $9(x - y)^2 - (x + 2y)^2 = [3(x - y)]^2 - (x + 2y)^2$
$= [(3x - 3y) + (x + 2y)][(3x - 3y) - (x + 2y)]$
$= (3x - 3y + x + 2y)(3x - 3y - x - 2y)$
$= (4x - y)(2x - 5y)$

(iii) $48x^3 - 27x = 3x(16x^2 - 9) = 3x[(4x)^2 - (3)^2] = 3x(4x + 3)(4x - 3)$

(iv) $16a^4 - b^4 = (4a^2)^2 - (b^2)^2 = (4a^2 + b^2)(4a^2 - b^2) = (4a^2 + b^2)(2a + b)(2a - b)$

(v) $(1 - x^2)(1 - y^2) + 4xy = 1 - x^2 - y^2 + x^2y^2 + 4xy = 1 + x^2y^2 + 2xy - x^2 - y^2 + 2xy$
$= (1 + x^2y^2 + 2xy) - (x^2 + y^2 - 2xy) = (1 + xy)^2 - (x - y)^2$
$= [(1 + xy) + (x - y)][(1 + xy) - (x - y)] = (1 + xy + x - y)(1 + xy - x + y)$

(vi) $(p^2 + q^2 - r^2)^2 - 4p^2q^2 = (p^2 + q^2 - r^2)^2 - (2pq)^2$
$= (p^2 + q^2 - r^2 + 2pq)(p^2 + q^2 - r^2 - 2pq)$
$= (p^2 + q^2 + 2pq - r^2)(p^2 + q^2 - 2pq - r^2) = [(p + q)^2 - r^2][(p - q)^2 - r^2]$

$= (p+q+r)(p+q-r)(p-q+r)(p-q-r)$

Q 5. Factorise:

(i) $25a^2 - 9b^2$
(ii) $a^2 - 81(b-c)^2$
(iii) $50a^3 - 2a$
(iv) $3a^5 - 108a^3$
(v) $a^4 - 1$
(vi) $(a+b)^3 - a - b$
(vii) $4a^2 - (4b^2 + 4bc + c^2)$
(viii) $9a^2 + 3a - 8b - 64b^2$
(ix) $4xy - x^2 - 4y^2 + z^2$
(x) $4x^2 - 12ax - y^2 - z^2 - 2yz + 9a^2$
(xi) $x^4 + x^2 + 1$
(xii) $(x^2 + 4y^2 - 9z^2)^2 - 16x^2y^2$
(xiii) $a^2 - b^2 - (a+b)^2$
(xiv) $x^2 + \frac{1}{x^2} - 11$
(xv) $4x^4 - x^2 - 12x - 36$

TYPE 5 – THE SUM AND DIFFERENCE OF TWO CUBES

We know $a^3 + b^3 = (a+b)^3 - 3ab\,(a+b) = (a+b)(a^2 - ab + b^2)$
Also, $a^3 - b^3 = (a-b)^3 + 3ab\,(a-b) = (a-b)(a^2 + ab + b^2)$

Therefore, Factors of $a^3 + b^3$ *are* $(a+b)$ and $(a^2 - ab + b^2)$
Factors of $a^3 - b^3$ *are* $(a-b)$ and $(a^2 + ab + b^2)$

Solved Examples

(i) Factorise: $a^3 + 27b^3$

$a^3 + 27b^3 = (a)^3 + (3b)^3$
$= (a+3b)[(a)^2 - a \text{ x } 3b + (3b)^2]$
$= (a+3b)(a^2 - 3ab + 9b^2)$

(ii) Factorise: $16a^4 + 54a$

$16a^4 + 54a = 2a\,(8a^3 + 27)$
$= 2a\,[(2a)^3 + (3)^3]$
$= 2a\,(2a+3)(4a^2 - 6a + 9)$

(iii) Factorise: $125a^3 + \frac{1}{8}$

$125a^3 + \frac{1}{8} = (5a)^3 + \left(\frac{1}{2}\right)^3$

$= \left(5a + \frac{1}{2}\right)[(5a)^2 - \left(5ax\ \frac{1}{2}\right) + \left(\frac{1}{2}\right)]^2$
$= \left(5a + \frac{1}{2}\right)\left(25a^2 - \frac{5}{2}a + \frac{1}{4}\right)$

(iv) Factorise: $a^3 + b^3 + a + b$

$a^3 + b^3 + a + b = (a + b)(a^2 - ab + b^2) + (a + b)$
$= (a + b)(a^2 - ab + b^2 + 1)$

(v) Factorise: $2a^7 - 128a$

$2a^7 - 128a = 2a\,(a^6 - 64)$
$= 2a[(a^3)^2 - (8)^2]$
$= 2a\,(a^3 + 8)(a^3 - 8)$
$= 2a\,(a^3 + 2^3)(a^3 - 2^3)$
$= 2a\,(a + 2)(a - 2)(a^2 - 2a + 4)(a^2 + 2a + 4)$

Q 6. Factorise:

(i) $a^3 - 27$
(ii) $64 - a^3b^3$
(iii) $3x^7y - 81x^4y^4$
(iv) $a^3 + 0.064$
(v) $(x - y)^3 - 8x^3$
(vi) $a^6 - b^6$
(vii) $a^3 - 27b^3 + 2a^2b - 6ab^2$
(viii) $a - b - a^3 + b^3$
(ix) $1029 - 3x^3$

Solved Examples

(i) Show that $15^3 - 8^3\, is\ divisible\ by\ 7$

Since $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$,

$15^3 - 8^3 = (15 - 8)(15^2 + 15 \text{ x } 8 + 8^2) = (7)(225 + 120 + 64) = 7 \text{ x } 409$, which is divisible by 7

(ii) Evaluate $\left(\frac{5.67 \text{ x } 5.67 \text{ x} 5.67 + 4.33\ x\ 4.33\ x\ 4.33}{5.67\ x\ 5.67 - 5.67\ x\ 4.33 + 4.33\ x\ 4.33}\right)$

Since $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

Therefore, $\frac{a^3 + b^3}{a^2 - ab + b^2} = (a + b)$ (I)

After resolving individual terms,

we get $\frac{(5.67)^3+(4.33)^3}{(5.67)^2-(5.67)(4.33)+(4.33)^2}$ = 5.67 + 4.33 =10

Q 7. Show that

(i) $13^3 - 5^3 \textit{is divisible by } 8$

(ii) $35^3 + 27^3 \text{is divisible by } 62$

Solved Examples

(i) Find the value of $(987)^2 - (13)^2$

We know $a^2 - b^2 = (a+b)(a-b)$

Let a = 987 and b = 13

Then $(987)^2 - (13)^2 = (987+13)(987-13) = 1000\ x\ 974 = 9{,}74{,}000$

Q 8. Find the value of

(i) $(67.8)^2 - (32.2)^2$

(ii) $\frac{(6.7)^2-(3.3)^2}{6.7-3.3}$

Solved Example

Factorise: $12(3x-2y)^2 - 3x + 2y - 1$

Let 3x – 2y = a

- $12(3x-2y)^2 - (3x-2y) - 1 = 12\,(a)^2 - a - 1$
- $12a^2 - 4a + 3a - 1 = 4a(3a-1) - 1(3a-1)$
- $[4\,(3x-2y) - 1][3\,(3x-2y) - 1]$
- $(12x - 8y - 1)(9x - 6y - 1)$

Q 9. Factorise

(i) $4\,(2x-3y)^2 - 8x + 12y - 2$

(ii) $3 - 5x + 5y - 12(x-y)^2$

(iii) $9x^2 + 3x - 8y - 64y^2$

END