GIRLS' HIGH SCHOOL AND COLLEGE, PRAYAGRAJ

WORKSHEET-4

CLASS IX (A,B,C,D, E &F)

SESSION 2020-2021

SUBJECT-MATHEMATICS

INSTRUCTIONS:- Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and website referred and thereafter answer the given questions.

Note:- 1. Student should refer to books of class 6, 7 & 8 for reference and also the following websites : www.extramarks.com and www.topperlearning.com

2. Concise MATHEMATICS I.C.S.E. Class-IX BY R.K. Bansal

3. Understanding I.C.S.E. MATHEMATICS class-IX By M.L. Aggarwal.

TOPIC :- RATIONAL AND IRRATIONAL NUMBERS

THE NUMBER SYSTEM

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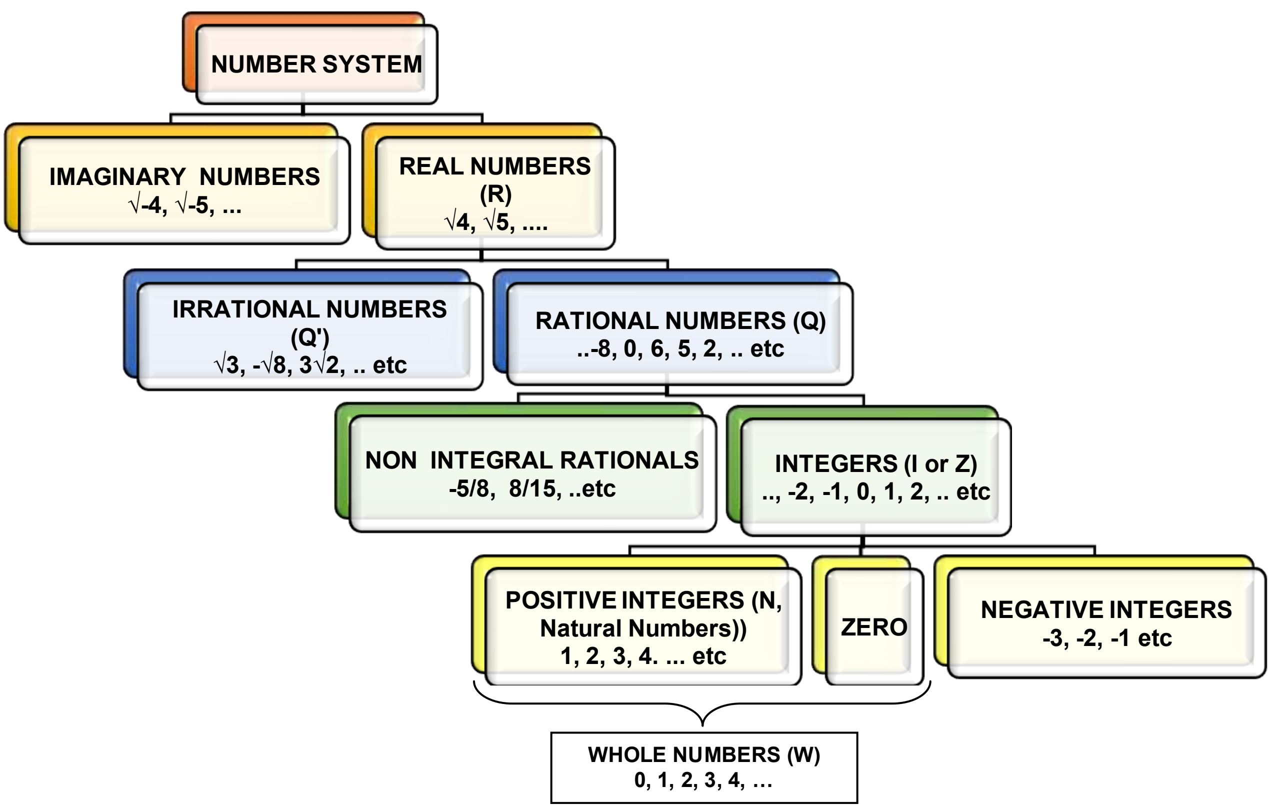

RATIONAL NUMBERS

IMPORTANT POINTS TO REMEMBER

1. $\frac{a}{b}$ is a rational number,
   - where $b \neq 0$
   - *a* and *b* do not have any common factor other than 1 (one) i.e. *a* and *b* are co-primes
   - *b* is usually positive whereas *a* may be negative, zero or positive

2. Every integer (positive, negative or zero) and every decimal number is a rational number.

3. Corresponding to every rational number $\frac{a}{b}$ its negative rational number is $\frac{-a}{b}$.
   Also, $-\frac{a}{b} = \frac{a}{-b} = \frac{-a}{b}$ and so on

4. (i) A rational number $\frac{a}{b}$, where a€I, b€I and b≠0, is positive if a and b both have the same sign. Thus, each of $\frac{5}{7}, \frac{-5}{-7}, \frac{-3}{-11}$ are all positive rational numbers.

   (ii) A rational number $\frac{a}{b}$ is negative if a and b have opposite signs.
   Thus, each of $-\frac{5}{7}, \frac{5}{-7}, \frac{-3}{11}$ are all negative rational numbers.

5. Two rational numbers $\frac{a}{b}$ *and* $\frac{c}{d}$ are equal if and only if a x d = b x c.
   i.e. $\frac{a}{b} = \frac{c}{d}$ *only if* $a \times d = b \times c$.
   Also, $\frac{a}{b} > \frac{c}{d}$, *then* $a \times d > b \times c$ *and if* $\frac{a}{b} < \frac{c}{d}$ *then* $a \times d < b \times c$.

6. For any two rational numbers a and b, $\frac{a+b}{2}$ is also a rational number which lies between a and b. Thus, a > b, then $a > \frac{a+b}{2} > b$ and if a < b, then $a < \frac{a+b}{2} < b$

Solved Example

Which of the rational numbers $\frac{3}{5}$ *and* $\frac{5}{7}$ is greater. Insert three rational numbers between $\frac{3}{5}$ *and* $\frac{5}{7}$ such that all the five numbers are in ascending order of their values.

- $\frac{3}{5}$ *and* $\frac{5}{7}$
- $\frac{3\text{x}7}{5\text{x}7}$ *and* $\frac{5\text{x}5}{7\text{x}5} = \frac{21}{35}$ *and* $\frac{25}{35}$
- Clearly $\frac{25}{35} > \frac{21}{35}$, Therefore $\frac{5}{7} > \frac{3}{5}$

Now. $\frac{3}{5} < \frac{5}{7}$ ⇒ $\frac{3}{5} < \frac{\left(\frac{3}{5}+\frac{5}{7}\right)}{2} < \frac{5}{7}$ ⇒ $\frac{3}{5} < \frac{23}{35} < \frac{5}{7}$

Now again $\frac{3}{5} < \frac{\left(\frac{3}{5}+\frac{23}{35}\right)}{2} < \frac{23}{35} < \frac{\left(\frac{23}{35}+\frac{5}{7}\right)}{2} < \frac{5}{7}$,

Therefore $\frac{3}{5} < \frac{22}{35} < \frac{23}{35} < \frac{24}{35} < \frac{5}{7}$ which are in ascending order.

Q 1 Solve the Following

(i) Arrange $-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3}$ *and* $\frac{11}{18}$ in the ascending order of their magnitude. Insert three rational numbers between $\frac{7}{12}$ *and* $\frac{11}{18}$

(ii) Is 'Zero' a rational number? Can it be written in the form of $\frac{p}{q}$, where p and q are integers and q≠0.

PROPERTIES OF RATIONAL NUMBERS

1. The sum of two or more rational numbers is always a rational number.

2. The difference of two or more rational numbers is always a rational number.
   i.e. If a and b are any two rational numbers, then each of a-b and b-a is also a rational number.

3. The product of two or more rational numbers is always a rational number.

4. The division of a rational number by a non-zero rational number is always a rational number.

DECIMAL REPRESENTATION OF RATIONAL NUMBERS

Terminating Decimals – The rational numbers whose decimal representation terminates. i.e. the division of numerator and denominator is exact and no remainder is left.
For example – $\frac{1}{8} = 0.125$ *or* $3\frac{2}{5} = 3.4$

Non Terminating Decimals – The division of a numerator to denominator never ends, no matter how long it continues. The quotients of such divisions are non-terminating decimals.
For example $\frac{3}{7} = 0.428571428 \ldots.or \frac{8}{23} = 0.7826086..$

Non Terminating Recurring Decimals – The process of division never ends and in the decimal part either a single digit or set of digits repeats in specific order.
For example $\frac{4}{9} = 0.44444444 \ldots.or \frac{4}{7} = 0.57142857142$

Note:- If the **denominator** of a rational number can be expressed as $\mathbf{2^m \times 5^n}$, where m and n are both whole numbers, the rational number is convertible into terminating decimal.

Solved Examples

Find whether each of the following is a terminating decimal or not.

(i) $\frac{17}{50}$

The denominator 50 can be written as $2 \text{ x } 5 \text{ x } 5 = 2^1 \text{ x } 5^2$, i.e. 50 can be expressed as 2^m x 5^n. Therefore, $\frac{17}{50}$ ***is a terminating decimal***

(ii) $\frac{23}{72}$

The denominator 72 can be written as $2 \text{ x } 2\text{x } 2 \text{ x } 3 \text{ x } 3 = 2^3 \text{ x } 3^2$, i.e. 72 cannot be expressed as 2^m x 5^n. Therefore, $\frac{23}{72}$ ***is not a terminating decimal***

Q 2 Solve the Following

Without doing any actual division, find which of the following rational numbers have terminating decimal representation.

(i) $\frac{7}{16}$ (ii) $\frac{9}{14}$ (iii) $\frac{43}{50}$ (iv) $\frac{123}{250}$ (v) $\frac{61}{75}$

IRRATIONAL NUMBERS (Q')

1. The square roots, cube roots etc of natural numbers are irrational numbers, if their exact values cannot be obtained. e.g. $\sqrt{m}$ is irrational if exact square root of m does not exist.

2. A non terminating and non recurring decimal is an irrational number. e.g. 0.4243445.., 3.862045…

3. The number π = $\frac{Circumference\ \ of\ a\ Circle}{Diameter\ \ of\ the\ circle\ \ taken}$ = 3.14159265358979…… We often take $\frac{22}{7}$ as an approximate value of π but π ≠ $\frac{22}{7}$.

Solved Example

Show that $\sqrt{2}$ is an irrational number.

We prove this by the **Method of Contradiction**

Let $\sqrt{2}$ be a rational number. Therefore $\sqrt{2} = \frac{a}{b}$ $or\ 2 = \frac{a^2}{b^2}$ $or\ a^2 = 2b^2$. This means a^2 is divisible by 2 and a is also divisible by 2. **(I)**

Let a = 2c, $a^2 = 4c^2$. Substituting for a^2 we get $2b^2 = 4c^2$ or $b^2 = 2c^2$. This means that b^2 is divisible by 2 and b is also divisible by 2. **(II)**

From **I & II** above we get, a and b are both divisible by 2 i.e. a and b have a common factor 2.

This contradicts our assumption that $\frac{a}{b}$ is rational i.e. a and b do not have any common factor other than unity (1).

Thus

- $\frac{a}{b}$ *is not a rational number*
- $\sqrt{2}$ *is not a rational number*,
- $\sqrt{2}$ *is an irrational number*

Similarly $\sqrt{3}, \sqrt{5}, \sqrt{6}, 3\sqrt{2}$ *etc can be proved as an irrational number*

Q 3 (A) Solve the Following

Show that $\sqrt{3}$ is an irrational number

Solved Example

Prove that $\sqrt{8} + 5$ *is irrational*

Let $\sqrt{8} + 5$ *is not irrational*, so $\sqrt{8} + 5$ *is a rational number*.
Now let $\sqrt{8} + 5 = x$, *a rational number*
$x^2 = (\sqrt{8} + 5)^2 = 8 + 25 + 2\sqrt{8} \text{ x } 5 = 33 + 10\sqrt{8} = 33 + 10 \text{ x} 2\sqrt{2}$ = 33 + 20$\sqrt{2}$

⇨ $x^2 = 33 + 20\sqrt{2}$ *or* $20\sqrt{2} = x^2 - 33$ *and* therefore $\sqrt{2} = \frac{x^2 - 33}{20}$ **(I)**

Since it is assumed that $\sqrt{8} + 5 = x$ *is rational*, x^2 *is rational and* $\frac{x^2 - 33}{20} = \sqrt{2}$ *is also rational*

But $\sqrt{2}$ *is irrational i.e.* x^2 *is irrational* **(II)**
From **I** we arrive at x^2 is rational
From **II** we arrive at x^2 is irrational

Therefore we arrive at a contradiction. So our assumption that $\sqrt{8} + 5$ *is rational is wrong*

Therefore, $\sqrt{8} + 5$ *is irrational*.

Q 3 (B) Solve the Following

Show that $3 - \sqrt{2}$ is irrational.

Solved Example

Find two irrational numbers between 2 and 3

Since 2 = $\sqrt{4}$ *and* $3 = \sqrt{9}$, therefore, each of $\sqrt{5}, \sqrt{6}, \sqrt{7}$ *and* $\sqrt{8}$ is an irrational number between 2 and 3.

Q 4 Solve the Following

(i) Insert two irrational numbers between 5 and 6.
(ii) Insert five irrational numbers between $2\sqrt{5}$ *and* $3\sqrt{3}$

Solved Example

Find two rational numbers between $\sqrt{2}$ *and* $\sqrt{3}$

Take any two rational numbers between 2 and 3 which are perfect squares i.e. 2.25, 2.56. 2.89.etc

Therefore, $\sqrt{2} < \sqrt{3}$ ⇨ $\sqrt{2} < \sqrt{2.25} < \sqrt{2.56} < \sqrt{3}$ ⇨ $\sqrt{2} < 1.5 < 1.6 < \sqrt{3}$

Q 5 Solve the Following

Write three rational numbers between $\sqrt{3}$ *and* $\sqrt{5}$

Solved Example

Identify each of the following as rational or irrational number.

(i) $(2+\sqrt{2})^2 = 2^2 + 2 \text{ x}2 \text{ x}\sqrt{2} + (\sqrt{2})^2 = 4 + 4\sqrt{2} + 2$

$= 6 + 4\sqrt{2}$ *which is an irrational number*

Therefore, $(2+\sqrt{2})^2$ *is an irrational number*

(ii) $\left(\frac{3}{2\sqrt{2}}\right)^2 = \frac{9}{4\text{x}2} = \frac{9}{8}$ *which is a rational number*

Therefore, $\left(\frac{3}{2\sqrt{2}}\right)^2$ *is a rational number*

Q 6 Solve the Following

A. State whether the following numbers are rational or not.

(i) $(3-\sqrt{3})^2$ (ii) $(5+\sqrt{5})(5-\sqrt{5})$ (iii) $(\sqrt{3}-\sqrt{2})^2$ (iv) $\left(\frac{\sqrt{7}}{6\sqrt{2}}\right)^2$

B. Find the square of

(i) $\left(\frac{3\sqrt{5}}{5}\right)$ (ii) $(\sqrt{3}+\sqrt{2})$ (iii) $(\sqrt{5}-2)$ (iv) $3 + 2\sqrt{5}$

Solved Example

Which of the following numbers is greater

(i) $3\sqrt{2}$ *or* $2\sqrt{3}$

$3\sqrt{2} = \sqrt{3^2 \text{x } 2} = \sqrt{18}$

$2\sqrt{3} = \sqrt{2^2 \text{x } 3} = \sqrt{12}$

Since $\sqrt{18} > \sqrt{12}$, we conclude that $3\sqrt{2} > 2\sqrt{3}$

(ii) $6\sqrt[3]{3}$ *and* $5\sqrt[3]{4}$

$6\sqrt[3]{3} = \sqrt[3]{6^3 x\, 3} = \sqrt[3]{648}$
$5\sqrt[3]{4} = \sqrt[3]{5^3 x\, 4} = \sqrt[3]{500}$
Since $\sqrt[3]{648} > \sqrt[3]{500}$, we conclude that $6\sqrt[3]{3} > 5\sqrt[3]{4}$

Q 7 Solve the Following

Write in ascending order

(i) $3\sqrt{5}$ *and* $4\sqrt{3}$ (ii) $2\sqrt[3]{5}$ *and* $3\sqrt[3]{2}$ (iii) $6\sqrt{5}$ *and* $7\sqrt{3}$

Solved Example

Compare $\sqrt[3]{4}$ *and* $\sqrt{3}$

$\sqrt[3]{4} = 4^{\frac{1}{3}}$ and $\sqrt{3} = 3^{\frac{1}{2}}$

Convert the powers into like fractions i.e. $\frac{1}{3} = \frac{1x\,2}{3x\,2} = \frac{2}{6}$ *and* $\frac{1}{2} = \frac{1x\,3}{2\,x\,3} = \frac{3}{6}$

$\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\frac{2}{6}} = (4^2)^{\frac{1}{6}} = 16^{\frac{1}{6}}$

$\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{3}{6}} = (3^3)^{\frac{1}{6}} = 27^{\frac{1}{6}}$

Clearly $27^{\frac{1}{6}} > 16^{\frac{1}{6}}$, therefore $\sqrt{3} > \sqrt[3]{4}$

Q 8 Solve the Following

A. Compare

(i) $\sqrt[6]{15}$ *and* $\sqrt[4]{12}$ (ii) $\sqrt{24}$ *and* $\sqrt[3]{35}$

B. Simplify each of the following

(i) $\sqrt[5]{16}\; x\; \sqrt[5]{2}$ (ii) $\frac{\sqrt[4]{243}}{\sqrt[4]{3}}$ (iii) $(3 + \sqrt{2})\,(4 + \sqrt{7})$ (iv) $(\sqrt{3} - \sqrt{2})^2$

IMPORTANT POINTS TO REMEMBER

1. For any two positive rational numbers x and y, if $\sqrt{x}$ *and* $\sqrt{y}$ are irrational, then if $\sqrt{x} > \sqrt{y}$ *implies* $x > y$ and $\sqrt{x} < \sqrt{y}$ *implies* $x < y$

2. If $a + b\sqrt{x} = c + d\sqrt{x},$ then a = c and b = d

3. The negative of an irrational number is always irrational.

4. The sum of a rational and irrational number is always irrational.

5. The product of a non-zero rational and irrational number is always irrational.

6. The sum, the difference, the product and the quotient of two irrational numbers need not be an irrational number.

SURDS (RADICALS)

If x is a positive rational number and n is an integer such that $x^{\frac{1}{n}}$, i.e. $\sqrt[n]{x}$ is irrational, then $x^{\frac{1}{n}}$ is called a surd or a radical. Examples $\sqrt{3}, \sqrt[4]{8},\ \sqrt[3]{20}$ *etc are all Surds*.

IMPORTANT POINTS

1. Every surd is an irrational number but every irrational number is not a surd. For example π is an irrational number but not a surd.

2. Let a be a rational number and n be a positive number greater than 1, Then $\sqrt[n]{a}$ *or* $a^{\frac{1}{n}}$ is called a surd of order n. Example $\sqrt{3}$ is a surd of order 2.

RATIONALIZATION

When two surds are multiplied together such that their product is a rational number, the two surds are called rationalizing factors of each other.

The process of rationalizing a surd by multiplying it with its rationalizing factor is called Rationalization.

Example (i) $5\sqrt{2} \text{ x } 3\sqrt{2} = 15 \text{ x } 2 = 30$ which is a rational number. Therefore $5\sqrt{2}$ *and* $3\sqrt{2}$ are rationalizing factors of each other.

(ii) $(3+\sqrt{5})(3-\sqrt{5}) = 3^2 - \sqrt{5}^2 = 9 - 5 = 4$ which is a rational number. Therefore $(3+\sqrt{5})$ *and* $(3-\sqrt{5})$ are rationalizing factors of each other.

Q 9 Solve the Following

Rationalize the denominator of

(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (ii) $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ (iii) $\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$ (iv) $\frac{3}{\sqrt{5}+\sqrt{2}}$

Solved Example

Find the value of a and b in the equation $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$

$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \text{x} \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(2+\sqrt{3})^2}{2^2-\sqrt{3}^2} = \frac{2^2+2\text{x} 2\text{x}\sqrt{3}+\sqrt{3}^2}{4-3} = \frac{4+4\sqrt{3}+3}{1} = 7+4\sqrt{3}$

Therefore, $a + b\sqrt{3} = 7 + 4\sqrt{3}$. This implies $\mathrm{a} = 7$ and $\mathrm{b} = 4$

Q 10 Solve the Following

Find the value of a and b in each of the following

(i) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + 6$ (ii) $\frac{3}{\sqrt{3}-\sqrt{2}} = a\sqrt{3} - b\sqrt{2}$

(iii) $\frac{5+3\sqrt{2}}{5-3\sqrt{2}} = a + b\sqrt{2}$

Solved Example

Simplify $\frac{22}{2\sqrt{3}+1} + \frac{17}{2\sqrt{3}-1}$

$\frac{22}{2\sqrt{3}+1} + \frac{17}{2\sqrt{3}-1} = \frac{22}{2\sqrt{3}+1} \text{x} \frac{2\sqrt{3}-1}{2\sqrt{3}-1} + \frac{17}{2\sqrt{3}-1} \text{x} \frac{2\sqrt{3}+1}{2\sqrt{3}+1} = \frac{22\,(2\sqrt{3}-1)}{(2\sqrt{3})^2-1^2} + \frac{17\,(2\sqrt{3}+1)}{(2\sqrt{3})^2-1^2} = \frac{44\sqrt{3}-22+34\sqrt{3}+17}{12-1}$

$= \frac{78\sqrt{3}-5}{11}$

Q 11 Solve the Following

A. Evaluate $\frac{4-\sqrt{3}}{4+\sqrt{3}} + \frac{4+\sqrt{5}}{4-\sqrt{5}}$

B. Show that $\frac{1}{3-2\sqrt{2}} - \frac{1}{2\sqrt{2}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5$

C. If $x = 2\sqrt{3} + 2\sqrt{2}, find\ the\ values\ of\ \frac{1}{x},\ \ x+\frac{1}{x},\ \ \left(x+\frac{1}{x}\right)^2$

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