Girls' High School &College, Prayagraj

Session: 2020 - 2021

SUBJECT: MATHS

CLASS : 6 (A,B,C,D,E,F)

WORKSHEET NO. 06

CHAPTER: Fundamental Concepts (Algebra)

NOTE: Parents ensure that the student takes a reference from the book of previous class or the internet. Following Links can be helpful in understanding the concepts :

https://youtu.be/e-ORvKIzBJs

https://youtu.be/T8XY_wL9G1o

Algebra:

Algebra is a generalized form of arithmetic. In Arithmetic, we use numbers like 5, -8, 0.64 etc., each with a definite value, whereas in Algebra, we use letters (*ab,c,.....x,y,z*) along with numbers. For example: *7x*, *3x – 2, 5a+b, 2y-7z* and so on.

The letters used in Algebra are called **variables** or **literal numbers** or simply **literals**.They do not have a fixed value.

SIGNS AND SYMBOLS:

In Algebra, the signs are used in the same sense as they are used in Arithmetic.

Also, the following *signs and symbols* are *frequently used in algebra*, each with the same meaning in every branch of mathematics.

=	means	*"is equal to"*	≠	means	*"is not equal to"*
<	means	*"is less than"*	>	means	*"is greater than"*
≮	means	*"is not less than"*	≯	means	*"is not greater than"*
∴	means	*"therefore"*	∵	means	*"because"* or *"since"*
~	means	*"difference between"*	⇒	means	*"implies that"*.

Writing a given statement in algebraic form:

Statement	Algebraic Form
(i) x subtracted from 8 is less than y	$8 - x < y$
(ii) y divided by 5 equals 2	$\frac{y}{5} = 2$
(iii) z increased by 2x is 23	$z + 2x = 23$

Conversely,

Algebraic Form		Statement
(i) $x + y = 3$		x plus y is equal to 3
	or	sum of x and y is equal to 3.
(ii) $p - 5 = x$		p minus 5 is equal to x
	or	p decreased by 5 is equal to x.
	or	p exceeds 5 by x
(iii) $5x > 7$		5 multiplied by x is greater than 7
	or	product of 5 and x is greater than 7
(iv) $\frac{8}{y} < 3$		8 divided by y is less than 3.

SOLVE THE FOLLOWING QUESTIONS :

1. **Express each of the following statements in algebraic form:**
   a) The sum of 8 and x is equal to y.
   b) z decreased by 3x is equal to y.
   c) 15 multiplied by m gives 3n.
   d) The sum of x and y is less than 24.
   e) 8y divided by x equal to 2z.
   f) The sum of 2 and x is greater than y.
2. **For each of the following algebraic expressions, write a suitable statement in words:**
   a) 3x+8=15
   b) 2y-x<12
   c) 5÷z=5
   d) (16+2a)-x>25
   e) 2x-3y=16

CONSTANTS AND VARIABLES:

There are two types of symbols in Algebra, namely **constants** and **variables**.

A symbol with a fixed numerical value in all situations is called a **constant** such as 5, 20, 456, -7, $\frac{5}{3}$, $\frac{7}{9}$, etc.

whereas a symbol whose value changes with situation is called a **variable** such as x, y, p, q, 5x, etc.

TERM:

A **term** is a constant or a variable or a product or a quotient of constants and variables. For example:

(i) 4 is a term, which is a *constant*

(ii) x is a term, which is a *variable*

(iii) 4x is a term, which is the *product of a constant and a variable.*

(iv) $\frac{3}{y}$ is a term, which is the *quotient of a constant and a variable.*

A term is called a **constant term** if it does not contain any literal (variable).

Thus, each of 3, –20, $\frac{5}{7}$, $-\frac{4}{9}$, etc. is a constant term.

A term is called a **constant term** if it does not contain any literal (variable). Thus each of 3, -20, 5/7, -4/9, etc. is a constant term.

Like Terms:

The terms having the same literal coefficient are called **like terms**. They may differ only in their numeral coefficients. For example:

a) xy, 5xy, -4xy, etc. are like terms.
b) $-8x^2y$, $7x^2y$, $1.5x^2y$, etc. are like terms.

Unlike Terms:

The terms that do not have the same literal coefficients are called **unlike terms**. For example:

a) 6b, 6ab, 6bc are unlike terms.
b) $2xy$, $2x^2y$ and $2xy^2$ are unlike terms.

ALGEBRAIC EXPRESSIONS :

An algebraic expression is a collection of one or more terms which are separated from each other by the signs + (plus) and/or – (minus).

For example :

Algebraic expressions	Number of terms used	Terms
(i) $5x$	1	$5x$
(ii) $8xy^2$	1	$8xy^2$
(iii) $3x + 8z$	2	$3x$ and $8z$
(iv) $4x - y + 7$	3	$4x$, y and 7
(v) $7xy + \frac{2a}{y} - 3z + 8$	4	$7xy$, $\frac{2a}{y}$, $3z$ and 8 and so on.

In the algebraic expression $4x - y + 7$, 7 is the constant term as it does not contain a literal.
Similarly, in the algebraic expression $7xy + \frac{2a}{y} - 3z + 8$; 8 is the constant term.

TYPES OF ALGEBRAIC EXPRESSIONS :

1) **Monomial :**
   An algebraic expression with only one term is called a Monomial. For Example: -8, z, xy, 2x, $\frac{3x}{5y}$ etc.
2) **Binomial :**
   An algebraic expression of two unlike terms is called a Binomial. For example: 5x+2y, 7-x, y+zy, $2a + \frac{b}{2}$, $\frac{a}{3} - \frac{b}{3}$, etc**.**

3) Trinomial :

An algebraic expression containing three unlike terms is called a Trinomial. For example: ax^2+bx+c , $2x^2-7x+4$, etc.

4) Multinomial :

An algebraic expression with two or more than two terms is called a Multinomial. For example:

Each of $3x+2$, $5-x$, a^2-7x is a multinomial of two terms.

$7+x-xy+y2$ is a multinomial of four terms and so on.

5) Polynomial:

An algebraic expression with one or more (unlike) terms is called Polynomial.

For example :

(i) Each of -20, 8, x, 5x, $3xy^2$, etc., is a polynomial.

(ii) $3x + 2y$ is a *polynomial of two terms.*

(iii) $x + 4yz - 7z + 8$ is a *polynomial of four terms.*

(iv) Every monomial, every binomial, every trinomial and every multinomial is a polynomial.

For each literal used in a polynomial, its power must always be a whole number.

(v) A polynomial can not be of the form : $\frac{1}{x}$, $\frac{3}{x+5}$, $\frac{2x}{x-5}$, $\frac{5}{x^2}$, $\frac{7x}{x^2+8}$, $x^{2/3}$, $x^{1/2}$, etc.

Terms are separated by plus (+) and minus (–) signs only.
The signs of multiplication (×) and division (÷) do not separate terms.
Thus, $3p + 5z - 7y$ has three terms, whereas $3p \times 5z - 7y$ has two terms only.
In the same way, $8 - 4x + 7y + 2z$ has four terms, whereas $8 \times 4x \times 7y \div 2z$ has only one term.

PRODUCTS AND FACTORS:

A **product** is the result of the multiplication of two or more constants or literals or of both. For example: 5xy is the product of 5, x and y.

Each constant and each literal multiplied together to form a product is called **factor** of that product.

COEFFICIENT :

Any factor for group of factors of a product is known as the **coefficient** of the remaining factors.

For example: **In the product 5axy,**

5 is the coefficient of axy, 5x is the coefficient of ay, xy is the coefficient of 5a, axy is the coefficient of 5 and so on.

If a factor is a numerical quantity it is called a **numeral coefficient** of the remaining factors, and if a factor involves letters, it is called a **literal coefficient** of the remaining factors.

For example: **In the product 3xy,**

3 is a numeral coefficient of xy, x is a literal coefficient of 3y, xy is a literal coefficient of 3, y is literal coefficient of 3x, 3y is literal coefficient of x and so on.

When the coefficient is unity, i.e. 1 (one) , it is usually omitted, i.e. 1b is written as b.

POWER OF LITERAL QUANTITIES:

When a quantity is multiplied by itself any number of times, the product is called *a power of that quantity*. This product is expressed by writing the number of like factors in it to the right of the quantity slightly raised.

For example :

$a \times a$ has 2 like factors, so to express it as : $a \times a = a^2$

Similarly, (i) $a \times a \times a$ has 3 like factors, so we write : $a \times a \times a = a^3$.

(ii) $a \times a \times a \times a \times a$ has 5 like factors, so we write : $a \times a \times a \times a \times a = a^5$.

The following table will make the concept, more clear :

	Product	Write as :	Read as :
(i)	$a \times a$	a^2	*a* squared or *a* raised to the power 2.
(ii)	$a \times a \times a$	a^3	*a* cubed or *a* raised to the power 3.
(iii)	$m \times m \times m \times m \times m$	m^5	*m* raised to the power 5 or fifth power of *m*.

In a^8, ***a*** is called the **base** and **8** is called the **exponent** or the **index** or the **power.**

Similarly, in x^5, **x** is the **base** and **5** is the **exponent** or the **index** or the **power** and so on.

1. For all values of x, $x^1 = x$ *i.e.* $5^1 = 5$, $8^1 = 8$, $35^1 = 35$ and so on

2. For all values of x, $x^0 = 1$ *i.e.* $5^0 = 1$, $8^0 = 1$, $35^0 = 1$ and so on

Example 1 :

Write each of the following products in **index form :**

(i) $m \times m \times n \times n \times n \times n$ (ii) $3 \times b \times b \times b \times b \times p \times p \times p$

Solution :

(i) $m \times m \times n \times n \times n \times n = \mathbf{m^2n^4}$ **(Ans.)**

(ii) $3 \times b \times b \times b \times b \times p \times p \times p = \mathbf{3b^4p^3}$ **(Ans.)**

Example 2 :

Write each of the following in **product form :**

(i) $3p^4$ (ii) $7b^2q^3$ (iii) $a^3m^4n^2$

Solution :

(i) $3p^4 = \mathbf{3 \times p \times p \times p \times p}$ **(Ans.)**

(ii) $7b^2q^3 = \mathbf{7 \times b \times b \times q \times q \times q}$ **(Ans.)**

(iii) $a^3m^4n^2 = \mathbf{a \times a \times a \times m \times m \times m \times m \times n \times n}$ **(Ans.)**

POLYNOMIAL IN ONE VARIABLE AND ITS DEGREE:

When an **algebraic expression** is **made of one variable** only, it is called a **polynomial in one variable.**

For example :

(i) $3 + 5x - 7x^2$ is a polynomial in variable x.

(ii) $9y^3 - 5y^2 + 8$ is a polynomial in variable y.

The **degree of a polynomial** in one variable **is the greatest** of the **exponents** (powers) **of its various terms.**

For example :

1. For polynomial $4x^2 - 3x^5 + 8x^6$
   (i) the exponent of the term $4x^2 = 2$,
   (ii) the exponent of the term $3x^5 = 5$ and
   (iii) the exponent of the term $8x^6 = 6$.
   Since the greatest exponent is 6
   ∴ The **degree** of the polynomial $4x^2 - 3x^5 + 8x^6 = \mathbf{6}$

The polynomial $3x^4 - x^3 + 5x - 7$ is in one variable only, the variable being x.
The polynomial $8y^5 - 3y^2 + 8$ is also in one variable only, the variable being y.

2. The **degree** of the polynomial $25 - x^4$ is **4.**
3. The **degree** of the polynomial $5x - 3$ is **1.**
4. The **degree** of the polynomial $4x^3 - 15x^5 - 7x^8$ is **8**

$\because x = x^1$

and so on.

Polynomials of two or more variables and their degree

For example :

(i) $3x + xy^2 - 8yz$ is a polynomial made of three variables, x, y and z.

(ii) $5y^3 - 3y^2x + 8x^2y^2 - 3x^5$ is a polynomial of two variables, x and y.

In order to find the degrees of such polynomials, find :

(a) The sum of the powers of all the variables used in each term of a given polynomial.

(b) The greatest of these sum is the degree of the given polynomial.

For example :

For polynomial $3x + xy^2 - 8yz$

The terms used are $3x$, xy^2 and $8yz$

Since the sum of the powers of the variables in $3x$ used = 1, $[3x = 3x^1]$

the sum of the powers of the variables in $xy^2 = 1 + 2 = 3$

and the sum of the powers of the variables used in $8yz = 1 + 1 = 2$

Clearly, **degree of the given polynomial = 3**

SOLVE THE FOLLOWING QUESTIONS :

1. **Separate the constants and the variables from each of the following:**

   7, 5x, -7y, $\frac{5}{3}$, $\frac{4}{5}$xy, az, 8p , 0 , $-\frac{xz}{3y}$

2. **Group the like terms together:**

   (i) 4x, -3y, -x, $\frac{2}{3}$x, $\frac{4}{5y}$, and y

   (ii) $-ab^2$, b^2a^2 , $7b^2a$, $-3a^2b^2$ and $2ab^2$

3. **State whether true or false:**

   (i) 15 is a constant and x is a variable, but 15x is variable.

   (ii) 16x has two terms 16 and x.

   (iii) 8 + ab is a binomial.

   (iv) The coefficient of y in – 4xy is -4.

   (v) The expression $2x^2$+x is a trinomial.

4. **State the number of terms in each of the following expressions:**

   (i) 2a-b (ii) 3x - $\frac{x}{p}$ (iii) 2x+y+8÷y (iv) xy ÷ 2

5. **State whether true or false:**

   (i) xy and -yx are like terms.

   (ii) -ba and 2ab are unlike terms.

   (iii) 5 and 5x are like terms.

   (iv) a and -a are like terms.

6. **For each expression given below, state whether it is monomial, or a binomial or a trinomial:**

   (i) xy (ii) 2x ÷ y (iii) 1 + x + y (iv) $ax^2 – x + 5$

SOLVE THE FOLLOWING QUESTIONS :

1. **Write down the coefficient of X in the following monomial:**
   (i) x (ii) – x (iii) - 5ax (iv) $\frac{3}{2}$ xy

2. **Write the coefficients of:**
   (i) x in $3xy^2$
   (ii) y in – y
   (iii) ax in $-axy^2$
   (iv) xy^2 in $5axy^2$

3. **State the numeral coefficient of the following monomials:**
   (i) 4xy (ii) abc (iii) $-\frac{2x}{y}$ (iv) - 7x ÷ y

4. **Write the degree of each of the following:**
   (i) $x + x^2$
   (ii) $5x^2 - 7x + 2$
   (iii) $x^3 - x^8 + x^{10}$
   (iv) $1 – 100x^2$
   (v) $8z^3 – 8y^2z^3 + 7yz^5$

END