GIRLS' HIGH SCHOOL AND COLLEGE

2020 – 2021

CLASS - 12 B & C

MATHEMATICS

WORKSHEET NO. 3

CHAPTER: MATRICES

Note: Parents are expected to ensure that the student spends two days to read and understand the topic according to the book or the website referred and thereafter answer the questions.

Book : ISc mathematics for class 12 by OP Malhotra

Website: www.khanacademy.org ,www.topperlearning.com or any other relevant website.

Principle of mathematical Induction:

Let P(n) be the statement involving positive integer n , such that

i. P(1) is true i.e. the statement is true for n = 1 and,
ii. P (m + 1) is true whenever P (m) is true, then P(n) is true for all positive integer n.

Ex. Let A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then A^n = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for every positive integer n.

Result can be proved by mathematical induction

Taking n = 1 , A = $\begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix}$

= $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ which is true for n = 1

Let the result be true for n= m then,

A^m = $\begin{bmatrix} 1+2m & -4m \\ m & 1-2m \end{bmatrix}$ now to show that the result is true for n = m + 1

A^{m+1} = $\begin{bmatrix} 1+2(m+1) & -4(m+1) \\ m+1 & 1-2(m+1) \end{bmatrix}$

As A^{m+1} = A^m A

= $\begin{bmatrix} 1+2m & -4m \\ m & 1-2m \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

= $\begin{bmatrix} 3+6m-4m & -4-8m+4m \\ 3m+1-2m & -4-1+2m \end{bmatrix}$

$$= \begin{bmatrix} 3+2m & -4-4m \\ m+1 & -1-2m \end{bmatrix}$$

$$= \begin{bmatrix} 1+2(m+1) & -4(m+1) \\ m+1 & 1-2(m+1) \end{bmatrix}$$

This shows that the result is true for n = m+ 1, whenever it is true for n =m. Hence by principle of mathematical induction, the result is valid for any positive integer n.

Solve the questions based on the above explanation:

1. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then prove that An= $\begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ for every positive integer n.
2. If A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$,prove that $(aI + bA)^n = a^n I + na^{n-1} bA$
3. Let A, B be two matrices such that they commute. Show that for any positive integer n
   i. $AB^n = B^nA$
   ii. $(AB)^n = A^nB^n$
4. If A = $\begin{bmatrix} cos\theta & isin\theta \\ isin\theta & cos\theta \end{bmatrix}$, then prove by mathematical induction that
   $A^n = \begin{bmatrix} cosn\theta & isin\, n\theta \\ isin\, n\theta & cosn\theta \end{bmatrix}$ for all n ϵ N
5. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, then use the principle mathematical induction to show that

   $A^n = \begin{bmatrix} 1 & n & n(n+1)/2 \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$ for every positive integer n.
6. If B and C are n rowed matrices and if A = B + C , BC = CB , $C^2 = 0$, then show that for every nϵ N, $A^{n+1} = B^n(B + (n+1)C)$.
7. If A = diag(a b c) , show that A^n = diag(a^n b^n c^n) for all positive integer n.

Questions based on the application of matrix multiplication:

1. A man buys 8 dozen mangoes, 10 dozen apples and 4 dozens bananas. Mangoes cost ₹18 per dozen, apples ₹9 per dozen and bananas ₹6 per dozen. Represent the quantities bought by a row matrix and the prices by a column matrix and hence obtain the total cost.
2. A store has in stock 20 dozen shirts, 15 dozen trousers and 25 dozen pairs of socks. If the selling prices are ₹50 per shirt, ₹90 per trousers and ₹12 per pair of

socks, then find the total amount the store owner will get after selling all the items in the stock.

3. A man invests ₹50,000 into two types of bonds. The first bond pays 5% interest per year and the second pays 6% interest per year. Using matrix multiplication, determine how to divide ₹50,000 among the two types of bonds so as to obtain an annual total interest of ₹2780.
4. There are two families A and B. There are 4 men, 6 women and 2 children in family A and 2 men, 2 women and 4 children in family B . The recommended daily allowance for calories is : Man: 24000, woman: 1900,child:1800 and for protein is : Man: 55gm, women: 45gm and child :33gm .
   Represent the above information by matrices. Using matrix multiplication , calculate the total requirement of calories and proteins for each of the two families.

5. The cooperative stores of a particular school has 10 dozen physics books, 8 dozen chemistry books and 5 dozen mathematics books. Their selling prices are ₹8.30, ₹3.42 and ₹4.50 each respectively. Find the total amount the store will receive from selling all the items.

Symmetric and Skew - Symmetric Matrix

A matrix is said to be **symmetric matrix** if it is symmetric about the principal diagonal for example: Let A =$\begin{bmatrix} \mathbf{4} & 3 \\ 3 & \mathbf{1} \end{bmatrix}$, where (**4 1**) is the principal diagonal and the elements above the principal diagonal and below the principal diagonal is same(i.e. 3) , another example ,

B= $\begin{bmatrix} \mathbf{1} & 2 & 3 \\ 2 & \mathbf{5} & 6 \\ 3 & 6 & \mathbf{7} \end{bmatrix}$ is symmetric , because $a_{12}= 2 = a_{21}$, $a_{13} = 3 = a_{31}$, $a_{23} = 6 = a_{32}$ i.e, $a_{ij} = a_{ji}$ for all i,j.

A **square** matrix A = [a_{ij}] is called a symmetric matrix, if (i, j)th element is same as (j , i)th element i.e., $a_{ij} = a_{ji}$. A necessary and sufficient condition for a matrix A to be symmetric is that it is equal to its transpose matrix .i.e., A' = A .

Diagonal matrices are always symmetric. (A square matrix whose all the elements are zero except for the principal diagonal is known as **diagonal matrix**.)

A **square** matrix which is symmetrical about the principal diagonal but with reverse sign is called as skew symmetric matrix. A square matrix A = [a_{ij}] is said to be **skew - symmetric** if the (i,j) th elements of A is the negative of the (j, i)th element of A ,i.e., if $a_{ij} = -a_{ji}$ for all i, j

Thus if A is skew symmetric matrix then $a_{ij} = -a_{ji}$ (by definition)

$\Rightarrow \quad a_{ii} =- a_{ii}$ for all values of i

$\Rightarrow \quad$ 2 aii = 0 $\Rightarrow a_{ii} = 0$ shows that $a_{11}= a_{22} = a_{33} = a_{44} = \dots a_{nn} = 0$

Thus each element of the principal diagonal is **zero.** In the matrix $\begin{bmatrix} \mathbf{0} & \mathbf{2} & \mathbf{3} \\ \mathbf{-2} & \mathbf{0} & \mathbf{1} \\ \mathbf{-3} & \mathbf{-1} & \mathbf{0} \end{bmatrix}$ the elements $a_{12} = -a_{21}$, $a_{13} = -a_{31}$, $a_{23} = -a_{32}$ and the principal diagonal is zero , such kind of matrix is known as skew symmetric matrix . Thus the necessary and sufficient condition for a skew symmetric matrix is A' = - A. A matrix is said to be **square null matrix** if it is both symmetric and skew symmetric matrix.

Solve the following questions :

1. Show that the elements on the main diagonal of a skew - symmetric matrix are all zero.
2. If A and B are symmetric matrices, then show that AB is symmetric iff AB = BA i.e., A and B commute.
3. Let A be a square matrix . Then show that

i. $A + A^t$ is a symmetric matrix.

ii. $A - A^t$ is a skew – symmetric matrix.

4. Show that B'AB is symmetric or skew symmetric according as A is symmetric or skew- symmetric.
5. Show that a matrix which is both symmetric as well as skew- symmetric is a null matrix.
6. Let A = $\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$. Find matrices X and Y such that X + Y = A , where X is a Symmetric and Y is skew- symmetric matrix.
7. Express the matrix A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew – symmetric matrix.{ Hint : Let P = $\frac{1}{2}$(A + A') and Q = $\frac{1}{2}$(A – A') show that P' = P and Q' = Q }
8. Show that A – A' is a skew symmetric matrix if

   i. $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

   ii. $\begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix}$

9. Find the symmetric and skew symmetric of the matrix.

   A = $\begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 3 \\ 3 & 5 & 7 \end{bmatrix}$

10. Show that all positive integral powers of a symmetric matrix are symmetric.