GIRLS' HIGH SCHOOL AND COLLEGE, PRAYAGRAJ

Session 2020-21

CLASS–X (A,B,C,D,E,F)

SUBJECT–MATHEMATICS

WORKSHEET NO. - 3

INSTRUCTIONS: – Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and websites given below.

NOTE – 1. Concise Mathematics ICSE Class X by R.K. Bansal

2. Understanding ICSE Mathematics Class X by M.L. Aggarwal

3. www.extramarks.com , www.topperlearning.com

Topic – Linear Inequations (In One Variable)

INTRODUCTION– If x and y are two quantities, then both of these quantities will satisfy any one of the following four conditions (relations) :

i.e. either (i) $x > y$ (ii) $x \geq y$ (iii) $x < y$ or (iv) $x \leq y$

Each of the four conditions given above, is an inequation

Linear Inequation in one variable :

If a, b and c are real number, then each of the following is called a linear inequation in one variable :

(i) $ax + b > c$. Read as : $ax + b$ is greater than c

(ii) $ax + b < c$. Read as : $ax + b$ is less than c

(iii) $ax + b \geq c$. Read as : $ax + b$ is greater than or equal to c

(iv) $ax + b \leq c$. Read as : $ax + b$ is less than or equal to c

In an inequation the signs '<' , '>' , '≤' , '≥' are called signs of inequality

Solving a linear Inequation Algebraically :

The following working rules must be adopted for solving a given linear inequation :

Rule 1: On transferring a positive term from one side of an inequation to its other side, the sign of the term becomes negative.

eg : $2x + 3 > 7$

$2x > 7 - 3$

Rule 2: On transferring a negative term from one side of an inequation to its other side, the sign of the term becomes positive.

eg : $5x - 4 \leq 15$

$5x \leq 15 + 4$

Rule 3: If each term of an inequation be multiplied or divided by the same positive number, the sign of inequality remains the same.

That is, if p is positive

eg $x < y \Rightarrow px < py$ and $\left[\frac{x}{p}\right] < \left[\frac{y}{p}\right]$

Rule 4: If each term of an inequation be multiplied or divided by the same negative number, the sign of inequality reverses.

That is, if p is negative

eg: $x < y \Rightarrow px > py$ and $\left[\frac{x}{p}\right] > \left[\frac{y}{p}\right]$

Rule 5: If sign of each term on both sides of an inequation is changed, the sign of inequality gets reversed.

eg: $-x > 5 \Leftrightarrow x < -5$

Rule 6: If both the sides of an inequation are positive or both are negative, then on taking their reciprocals, the sign of inequality reverses.

ie: If x and y both are either positive or both are negative, then

ie: $x > y \Leftrightarrow \frac{1}{x} < \frac{1}{y}$

REPLACEMENT SET AND SOLUTION SET

The set, from which the value of the variable x is to be chosen, is called replacement set and its subset, whose elements satisfy the given inequation, is called solution set.

eg : Let the given inequation be $x < 3$, if :

the replacement set = N, the set of natural numbers,

The solution set = {1,2}

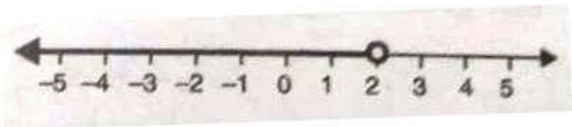
But, if the replacement set is the set of real numbers, the solution set can only be described in set builder form, ie $\{x: \ x \in R \ and \ x < 3\}$

REPRESENTATION OF THE SOLUTION ON THE NUMBER LINE

A real number line can be used to represent the solution set of an inequation

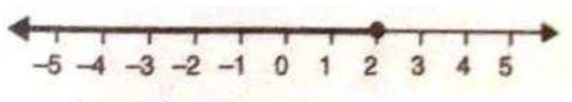
For eg : The adjacent figure shows

$x < 2$ and $x \in R$



The number 2 is encircled and the circle is not darkened to show that 2 is not included in the graph.

If 2 is also included ie $x \leq 2$, then circle will be darkened and the graph will be as shown along side:



Example 1: Given that $x \in R$, solve the following inequality and graph the solution on the number line : $-1 \leq 3 + 4x < 23$

Solution :

Given : $-1 \leq 3 + 4x < 23; \ \ x \in R$

$$-1 \leq 3 + 4x \ \ and \ \ 3 + 4x < 23 - 3$$

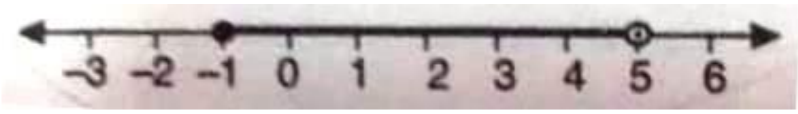
$\Rightarrow -4 \leq 4x \;\; and \;\; 4x < 20$

$\Rightarrow \;\; -1 \leq x \;\; and \;\; x < 5$

$\Rightarrow \;\; -1 \leq x < 5 \;\; ; \;\; x \in R$

Solution = $\{-1 \leq x < 5; x \in R\}$

Solution on the number line is :



Example 2 : Given $P = \{x: \;\; 5 < 2x - 1 \leq 11, \; x \in R\}$

$Q = \{x: -1 \leq 3 + 4x < 23, x \in I\}$

Where R = {real numbers} and I = {integers}

Represent P and Q on two different number lines. Write down the elements of P and Q

Solution :

For P : $5 < 2x - 1 < 11, \quad where \;\; x \in R$

$\Rightarrow \;\; 5 < 2x - 1 \;\; and \;\; 2x - 1 \leq 11$

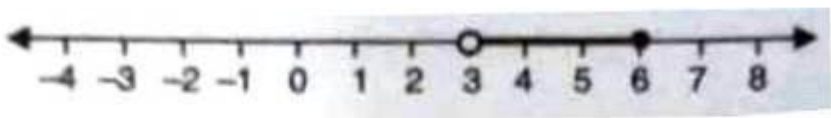
$\Rightarrow \;\; 5 + 1 < 2x \;\; and \;\; 2x < 11 + 1$

$\Rightarrow \frac{6}{2} < x \quad and \;\; 2x < 12$

$= x < \frac{12}{2}$

$\Rightarrow 3 < x \qquad x < 6$

ie $3 < x \leq 6 \, ; x \in R$

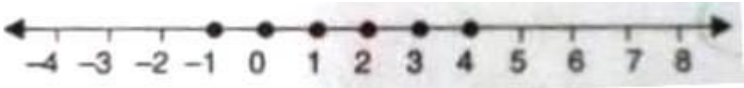
$\therefore$ P = 

For Q : $-1 \leq 3 + 4x < 23 \, ; x \in I$

$-1 \leq 3 + 4x \quad and \quad 3 + 4x < 23$

$-1 \le x \quad and\ x < 5$

ie $-1 \le x < 5$; $\quad where\ x \in I$

Q = 

Hence $P \cap Q$ = {elements common to both P and Q}

= {4}

Example 3 : Find three smallest consecutive whole numbers such that the difference between one-fourth of the largest and one fifth of the smallest is at least 3

Solution : Let the required whole numbers $x, x+1\ \ and\ \ x+2$

According to given statement :

$\Rightarrow \frac{x+2}{4} - \frac{x}{5} \ge 3$

$\Rightarrow \frac{5x+10-4x}{20} \ge 3$

$\Rightarrow x + 10 \ge 60$

$\Rightarrow x \ge 50$

Since, the smallest value of x = 50 that satisfies the inequation $x \ge 50$

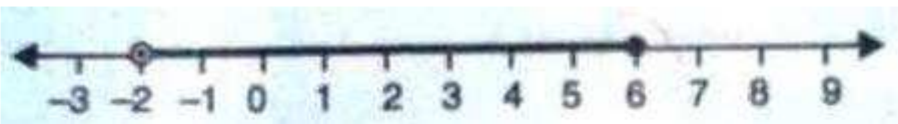
∴ Required smallest consecutive whole numbers are :

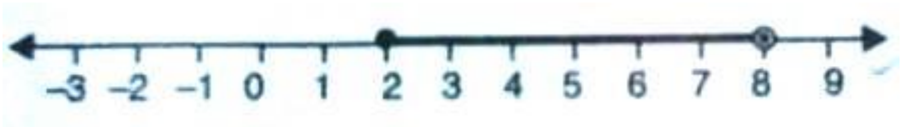
$x, x+1\ \ and\ \ x+2$

50, 50 +1 and 50 +2

50, 51 and 52

Example 4 : The diagram, given below, represents two inequations P and Q on real number lines :

P = 

Q = 

(i) Write down P and Q in set builder notation.

(ii) Represent each of the following sets on different number lines :

(a) P ∪ Q (b) P ∩ Q (c) P – Q

(d) Q – P (e) P ∩ Q' (f) P' ∩ Q

Solution :

(i) $P = \{x: -2 < x \leq 6 \ \ and\ x \in R\}\ and$

$Q = \{x : 2 \leq x < 8 \ \ and\ x \in R\}$

(ii) (a) P ∪ Q = Numbers which belong to P or to Q or to both

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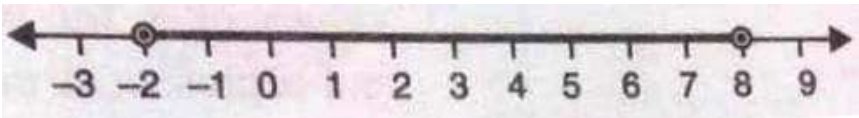

(b) P ∩ Q = Numbers common to both P and Q

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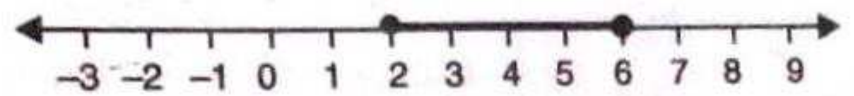

(c) P – Q = Numbers which belong to P but do not belong to Q

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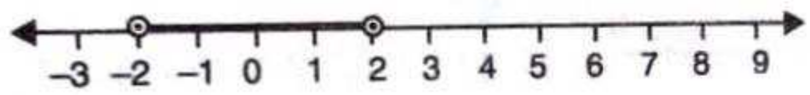

(d) Q – P = Numbers which belong to Q but do not belong to P

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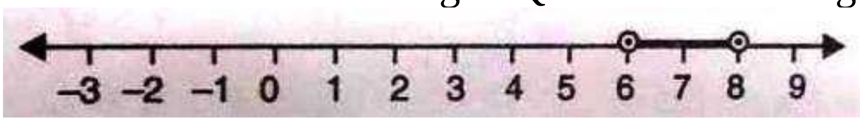

(e) P ∩ Q' = Numbers which belong to P but do not belong to Q = P – Q

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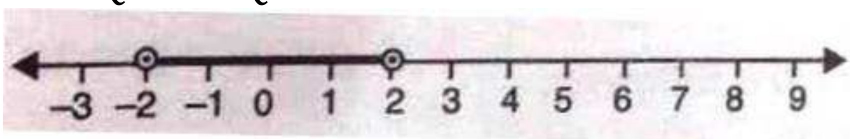
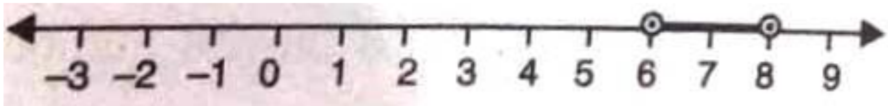


(f) P' ∩ Q = Numbers which do not belong to P but belong to Q

= Q – P

= 

SOLVE THE FOLLOWING QUESTIONS:

Question 1 – Find the range of values of x, which satisfy:

$$-\frac{1}{3} \leq \frac{x}{2} + 1\frac{2}{3} < 5\frac{1}{6}$$

Graph, in each of the following cases, the values of x on the different real number lines:

(i) $x \in W$ (ii) $x \in Z$ (iii) $x \in R$

Question 2 – Given : $A = \{x : -8 < 5x + 2 \leq 17, x \in I\}$

$$B = \{x : -2 \leq 7 + 3x < 17, x \in R\}$$

Where R = {real numbers} and I = {integers}

Represent A and B on two different number lines. Write down the elements of A ∩ B.

Question 3 – Solve the following inequation and represent the solution set on the number line $2x - 5 \leq 5x + 4 < 11$, where $x \in I$.

Question 4 – Given that $x \in I$, solve the inequation and graph the solution on the number line :

$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$$

Question 5 – Given :

$A = \{x : 11x - 5 > 7x + 3, x \in R\}$ and

$B = \{x : 18x - 9 \geq 15 + 12x, \ x \in R\}$

Find the range of set A ∩ B and represent it on a number line

Question 6 – Find the set of values of x, satisfying :

$$7x + 3 \geq 3x - 5 \ and \ \frac{x}{4} - 5 \leq \frac{5}{4} - x, \ where \ x \in N$$

Question 7 – Solve :

(i) $\frac{x}{2} + 5 \leq \frac{x}{3} + 6$, where x is a positive odd integer

(ii) $\frac{2x+3}{3} \geq \frac{3x-1}{4}$, where x is a positive even integer

Question 8 – Solve the inequation :

$$-2\frac{1}{2} + 2x \leq \frac{4x}{5} \leq \frac{4}{3} + 2x, \ \ x \in W$$

Graph the solution set on the number line.

Question 9 – Find three consecutive largest positive integers such that the sum of one-third of first, one-fourth of second and one-fifth of third is at most 20.

Question 10 – Solve the given inequation and graph the solution on the number line.

$$2y - 3 < y + 1 \leq 4y + 7, y \in R$$

Question 11 – Solve the following inequation and represent the solution set on the number line.

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in R$$

Question 12 – Solve the following inequation and represent the solution set on the number line :

$$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, x \in R$$

Question 13 – Solve the following inequation, write the solution set and represent it on the number line:

$$-\frac{x}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6},\ x \in R$$

Question 14 – Find the value of x, which satisfy the inequation

$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2, x \in W$. Graph the solution set on the number line.

Question 15 – Solve the following inequation and write the solution set:

$$13x - 5 < 15x + 4 < 7x + 12,\ x \in R$$

Represent the solution on a real number line.

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