GIRLS' HIGH SCHOOL AND COLLEGE, PRAYAGRAJ

WORKSHEET :- 3

SESSION :- 2020-21

CLASS :- IX   A, B, C, D, E, F

SUBJECT:- MATHEMATICS

INSTRUCTIONS :- Parents are expected to ensure that the student spends two days to read and understand the chapter according to the books and website referred and thereafter answer the given questions.

Note :- 1. Student should refer to books of class 6, 7 & 8 for reference and also the following websites :" www.extramarks.com " and "www.topperlearning.com"

2.Concise MATHEMATICS I.C.S.E. Class -IX By R.K. Bansal

3.Understanding I.C.S.E. MATHEMATICS class -IX By M.L. Aggarwal

TOPIC :- EXPANSIONS

FORMULAE :

1.$(x+y)^2 = x^2 + 2xy + y^2$.

2.$(x-y)^2 = x^2 - 2xy + y^2$

3.$(x+y)(x-y) = x^2 - y^2$.

4.$(x+\frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$.

5.$(x-\frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$.

6.$\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right) = x^2 - \frac{1}{x^2}$.

7. $(x+\frac{1}{x})^2 + (x-\frac{1}{x})^2 = 2\left(x^2 + \frac{1}{x^2}\right)$.

8. $(x+\frac{1}{x})^2 - (x-\frac{1}{x})^2 = 4$.

9.$(x+a)(x+b) = x^2 + (a+b)x + ab$.

10.$(x + a)(x - b) = x^2 + (a - b)x - ab$.

11.$(x - a)(x + b) = x^2 - (a - b)x - ab$.

12.$(x - a)(x - b) = x^2 - (a + b)x + ab$.

13.$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$.

14.$(x + y)^3 = x^3 + y^3 + 3x^2y + 3xy^2 = x^3 + y^3 + 3xy(x + y)$.

15.$(x - y)^3 = x^3 - y^3 - 3x^2y + 3xy^2 = x^3 - y^3 - 3xy(x - y)$.

16. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$.

17. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

18.$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + y^3 + z^3 - 3xyz$.

19.$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$.

20.$If\ a + b + c = 0, then\ a^3 + b^3 + c^3 = 3abc$.

Important points:

1. The expression $x^2 + 2xy + y^2$ is called the expansion of $(x + y)^2$.

2. The above results from 1 to 3 and 9 to 19 are true for all values of the variables involved.

3. Results from 4 to 8 are true for all values of x except 0.

4. An equation which is true for all values of the variables involved is called an identity.

EXAMPLE:

1.Evaluate: $(2x + 3y)^2$

Solution: $(2x + 3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$

$$= 4x^2 + 12xy + 9y^2.$$

2. Find the expansion of $(2a + 3b - 4c)^2$.

Solution: $(2a + 3b - 4c)^2$= $[2a + 3b + (-4c)]^2$

$= (2a)^2 + (3b)^2 + (-4c)^2 + 2[(2a)(3b) + (3b)(-4c) + (-4c)(2a)]$

$= 4a^2 + 9b^2 + 16c^2 + 2(6ab - 12bc - 8ca)$

$= 4a^2 + 9b^2 + 16c^2 + 12ab - 24bc - 16ca.$

3. Find the expansion of $\left(\frac{2}{3}x + \frac{5}{7}y\right)^2$.

Solution: $\left(\frac{2}{3}x + \frac{5}{7}y\right)^2$= $\left(\frac{2}{3}x\right)^2 + 2\left(\frac{2}{3}x\right)\left(\frac{5}{7}y\right) + \left(\frac{5}{7}y\right)^2$

$= \frac{4}{9}x^2 + \frac{20}{21}xy + \frac{25}{49}y^2.$

4.Find the expansion of $\left(\frac{1}{2}x - \frac{2}{3}y - \frac{4}{5}z\right)^2$.

Solution: $\left(\frac{1}{2}x - \frac{2}{3}y - \frac{4}{5}z\right)^2 = \left(\frac{1}{2}x + \left(-\frac{2}{3}y\right) + \left(-\frac{4}{5}z\right)\right)^2$

$=\left(\frac{1}{2}x\right)^2 + \left(-\frac{2}{3}y\right)^2 + \left(-\frac{4}{5}z\right)^2 + 2\left[\left(\frac{1}{2}x\right)\left(-\frac{2}{3}y\right) + \left(-\frac{2}{3}y\right)\left(-\frac{4}{5}z\right) + \left(-\frac{4}{5}z\right)\left(\frac{1}{2}x\right)\right]$

$=\frac{1}{4}x^2 + \frac{4}{9}y^2 + \frac{16}{25}z^2 + 2\left[-\frac{1}{3}xy + \frac{8}{15}yz - \frac{2}{5}zx\right]$

$=\frac{1}{4}x^2 + \frac{4}{9}y^2 + \frac{16}{25}z^2 - \frac{2}{3}xy + \frac{16}{15}yz - \frac{4}{5}zx.$

5. Simplify : $(2x - 3y + 5)(2x - 3y - 5)$

Solution: Given expression=$(2x - 3y + 5)(2x - 3y - 5)$.

Let 2x-3y=a, then

Given expression = $(a + 5)(a - 5) = a^2 - 5^2 = a^2 - 25$

$= (2x - 3y)^2 - 25$

$= (2x)^2 - 2(2x)(3y) + (3y)^2 - 25$

$= 4x^2 - 12xy + 9y^2 - 25.$

6.Simplify: $\left(2p-\frac{q}{5}-3\right)\left(2p+\frac{q}{5}+3\right)$

Solution: Given expression= $\left(2p-\frac{q}{5}-3\right)\left(2p+\frac{q}{5}+3\right)$

$$=\left\{2p-\left(\frac{q}{5}-3\right)\right\}\left(2p+\left(\frac{q}{5}+3\right)\right)$$

$$=(2p)^2-\left(\frac{q}{5}+3\right)^2$$

$$=4p^2-\left\{\left(\frac{q}{5}\right)^2+2\left(\frac{q}{5}\right)(3)+(3)^2\right\}$$

$$=4p^2-\frac{q^2}{25}-\frac{6q}{5}-9.$$

7.Find the expansion of $(2a+3b)^3$

Solution: $(2a+3b)^3$ = $(2a)^3+(3b)^3+3(2a)(3b)(2a+3b)$

$$=8a^3+27b^3+18ab(2a+3b)$$

$$=8a^3+27b^3+36a^2b+54ab^2.$$

8.Find the expansion of $(x+y-1)^3$

Solution: $(x+y-1)^3$ = $(\overline{x-y}-1)^3$

$$=(x+y)^3-(1)^3-3(x+y)(1)(\overline{x+y}-1)$$

$$=(x+y)^3-1-3(x+y)(\overline{x+y}-1)$$

$$=[x^3+y^3+3xy(x+y)]-1-3(x+y)^2+3(x+y)$$

$$=x^3+y^3+3x^2y+3xy^2-1-3(x^2+2xy+y^2)+3x+3y$$

$$=x^3+y^3+3x^2y+3xy^2-3x^2-6xy-3y^2+3x+3y-1.$$

9.Find the product of $(2x+5y+3)(2x+5y+4)$

Solution: Given expression = $(2x+5y+3)(2x+5y+4)$

Let 2x+5y=a, then

.: Given expression = $(a+3)(b+4)$

$$= a^2 + (3+4)a + 3 \times 4$$

$$= a^2 + 7a + 12$$

$$= (2x+5y)^2 + 7(2x+5y) + 12$$

$$= (2x)^2 + 2(2x)(5y) + (5y)^2 + 14x + 35y + 12$$

$$= 4x^2 + 20xy + 25y^2 + 14x + 35y + 12.$$

10. Simplify: $(3x+5y)(9x^2 - 15xy + 25y^2)$

Solution: We know that $(a+b)(a^2 - ab + b^2) = a^3 + b^3$

Given expression = $(3x+5y)[(3x)^2 - (3x)(5y) + (5y^2)$

$$= (3x)^3 + (5y)^3$$

$$= 27x^3 + 125y^3.$$

11. Simplify: $(x+3y+5z)(x^2 + 9y^2 + 25z^2 - 3xy - 15yz - 5zx)$.

Solution: We know that

$$(x+y+z)(x^2+y^2+z^2-xy-yz-zx) = x^3+y^3+z^3-3xyz$$

.:Given expression = $(x+3y+5z)[(x)^2 + (3y)^2 + (5z)^2 - (x)(3y) - (3y)(5z) - (5z)(x)]$

= $(x)^3 + (3y)^3 + (5z)^3 - 3(x)(3y)(5z)$

= $x^3 + 27y^3 + 125z^3 - 45xyz.$

12. Find the coefficient of x^2 and x in the product of $(x-5)(x+3)(x+7)$.

Solution: We know that

$$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc.$$

Compare $(x-5)(x+3)(x+7)$ with $(x+a)(x+b)(x+c)$

Here $a = -5, b = 3 \; and \; c = 7$

.: Coefficient of $x^2 = a + b + c = -5 + 3 + 7 = 5$ and

Coefficient of $x = ab + bc + ca = (-5) * 3 + 3 * 7 + 7 * (-5) = -29$

13. By using $(x+y)^2 = x^2 + 2xy + y^2$, find the value of $(10.3)^2$

Solution. $(10.3)^2 = (10 + 0.3)^2$

$= (10)^2 + 2 * 10 * 0.3 + (0.3)^2$

$= 100 + 6 + .09$

$= 106.09$

14. If $a + b + c = 0$, prove that $a^3 + b^3 + c^3 = 3abc$.

Solution. Given $a + b + c = 0$ implies $a + b = -c$

On cubing both sides, we get $(a+b)^3 = (-c)^3$

Implies $a^3 + b^3 + 3ab(a+b) = -c^3$

But $a + b = -c$

Therefore, $a^3 + b^3 + 3ab(-c) = -c^3$

Implies $a^3 + b^3 + c^3 - 3abc = 0$

Implies $a^3 + b^3 + c^3 = 3abc$.

15. If $x^4 + \frac{1}{x^4} = 194$, find the values of :

(i) $x^2 + \frac{1}{x^2}$ (ii) $x + \frac{1}{x}$ (iii) $x^3 + \frac{1}{x^3}$

Solution. (i) $\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$

$= 194 + 2 = 196$

Implies $x^2 + \frac{1}{x^2} = \pm 14$ but $x^2 + \frac{1}{x^2}$ is always positive,

$\therefore x^2 + \frac{1}{x^2} = 14$

(ii) $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = 14 + 2$

Implies $\left(x + \frac{1}{x}\right)^2 = 16$

Implies $x + \frac{1}{x} = \pm 4$

(iii) $\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3*x*\frac{1}{x}\left(x+\frac{1}{x}\right)$

$=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

Implies $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)$.

Two cases arise:

Case I. When $x+\frac{1}{x}=4$,

$x^3+\frac{1}{x^3}=4^3-3*4=64-12=52$

Case II. When $x+\frac{1}{x}=-4$

$x^3+\frac{1}{x^3}=(-4)^3-3*(-4)=-64+12=-52.$

EXERCISE

By using standard formulae, expand the following (1 to 9):

1. $(2x+7y)^2$
2. $\left(3x+\frac{1}{2x}\right)^2$
3. $\left(3x-\frac{1}{2x}\right)^2$
4. $(x+3)(x+1)$
5. $(x-2y-z)^2$
6. $\left(2x+\frac{3}{x}-1\right)^2$
7. $(x+2)^3$
8. $\left(3x+\frac{1}{x}\right)^3$
9. $(5x-3y)^3$

Simplyfy the following:

10. $(a+b)^2+(a-b)^2$

11. If *x – y = 8* and *xy = 5*, find x^2+y^2.

12. If *x + y = 10* and *xy = 21*, find $2(x^2+y^2)$.

13. If *2a + 3b = 7* and *ab = 2*, find $4a^2+9b^2$.

14.If $3x - 4y = 16$ and $xy = 4$, find the value of $9x^2 + 16y^2$.

15.If $x + y = 8$ and $x - y = 2$, find the value of $2x^2 + 2y^2$.

16.If $a^2 + b^2 = 13$ and ab = 16,find (i) $a + b$ (ii) $a - b$

17.If a + b = 4 and ab = -12, find (i) a-b (ii) $a^2 - b^2$.

18.If p - q = 9 and pq = 36, evaluate

(i) p + q (ii) $p^2 - q^2$.

19.If x + y = 6 and x - y = 4, find (i) $x^2 + y^2$ (ii) xy.

20.If $x - 3 = \frac{1}{x}$, find the value of $x^2 + \frac{1}{x^2}$.

21. If $x + y = 8$ and $xy = 3\frac{3}{4}$ find the value of

(i) $x - y$ (ii) $3(x^2 + y^2)$ (iii) $5(x^2 + y^2) + 4(x - y)$.

22. If $a - b = 3$ and $ab = 4$, find the value of $a^3 - b^3$.

23. If $x + \frac{1}{x} = 4$, find the value of

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$ (iii) $x^3 + \frac{1}{x^3}$ (iv) $x - \frac{1}{x}$

24. If $x - \frac{1}{x} = \sqrt{5}$, find the values of

(i) $x^2 + \frac{1}{x^2}$ (ii) $x + \frac{1}{x}$ (iii) $x^3 + \frac{1}{x^3}$

25. If $x + \frac{1}{x} = 2$, Prove that $x^2 + \frac{1}{x^2} = x^3 + \frac{1}{x^3} = x^4 + \frac{1}{x^4}$

26. If a + 2b = 5, prove that $a^3 + 8b^3 + 30ab = 125$

27. If $x^2 + \frac{1}{x^2} = 27$, find the value of $x - \frac{1}{x}$.

28. If $x^2 + \frac{1}{25x^2} = 8\frac{3}{5}$, find $x + \frac{1}{5x}$

29. If $a^2 - 3a + 1 = 0$, find (i) $a^2 + \frac{1}{a^2}$ (ii) $a^3 + \frac{1}{a^3}$

30. If $a + b + c = 12$ and $ab + bc + ca = 22$, find $a^2 + b^2 + c^2$.

31. If $a^2 + b^2 + c^2 = 125$ and $ab + bc + ca = 50$, find $a + b + c$.

32. If $a - b = 7$ and $a^2 + b^2 = 85$, then find the value of $a^3 - b^3$.

33. If the sum and the product of two numbers are 8 and 15 respectively, find the sum of their cubes.

34. If the number x is less than the number y and the sum of the squares of x and y is 29, find the product of x and y.

35. $(3x - 1)^2 - (3x - 2)(3x + 1)$

36. $(7p + 9q)(7p - 9q)$

37. $(2x - y + 3)(2x - y - 3)$

38. $[x + \frac{2}{x} - 3]\ [x - \frac{2}{x} - 3]$

39. $(x + 2y + 3)(x + 2y + 7)$

40. $(2p + 3q)(4p^2 - 6pq + 9q^2)$

41. $(3p - 4q)(9p^2 + 12pq + 16q^2)$

42. $(2x + 3y + 4z)(4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8zx)$

43. Find the product of the following:

(i) $(x + 1)\ (x + 2)\ (x + 3)$

(ii) $(x - 2)\ (x - 3)\ (x + 4)$

44. Find the coefficient of x^2 and x in the product of $(x - 3)(x + 7)(x - 4)$.

45. If $a^2 + 4a + x = (a + 2)^2$, find the value of x.

46. Use $(a + b)^2 = a^2 + 2ab + b^2$ to evaluate the following:

$(i)\ (101)^2$ $(ii)\ (1003)^2$ $(iii)\ (10.2)^2$

47. Use $(a - b)^2 = a^2 - 2ab + b^2$ to evaluate the following:

$(i)\ (99)^2$ $(ii)\ (997)^2$ $(iii)\ (9.8)^2$

48. By using suitable identities, evaluate the following:

(i) $(103)^3$ (ii) $(99)^3$ (iii) $(10.1)^3$.

49. If $2a - b + c = 0$, prove that $4a^2 - b^2 + c^2 + 4ac = 0$.

50. If $a + b + 2c = 0$ prove that $a^3 + b^3 + 8c^3 = 6abc$.

THE END